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Narong Tanawongnukul
Iowa State University

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**The mechanism to forecast competitors' production costs and to adjust a supplier's
sealed bid to win a short-term profit on the margin in the power auction**

by

Narong Tanawongnukul

A thesis submitted to the graduate faculty
in partial fulfillment of the requirements for the degree of
MASTER OF SCIENCE

Major: Industrial Engineering

Program of Study Committee
John Jackman (Major Professor)
Jo Min
Troy Strader
Jinhua Zhao

Iowa State University

Ames, Iowa

2002

Graduate College
Iowa State University

This is certify that the master's thesis of

Narong Tanawongnukul

has met the thesis requirements of Iowa State University

Signatures have been redacted for privacy

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ABSTRACT

The U.S. electric power industry has experienced many changes in the last ten years due to deregulation and opened to competition. The formation of electricity in California and in many parts of the world is based on a sealed bid auction mechanism. In a sealed bid auction, each supplier has no knowledge of other suppliers' bids. The primary of this study is to investigate a mechanism for increasing a supplier's short-term profitability and the number of winning auctions. The new mechanism is based on an ARIMA model in order to forecast other suppliers' production costs in this study. The new mechanism is proposed to adjust the supplier's offered bid to win on the margin in the auction. This study assumes that the majority of production cost for each supplier is from the coal cost. Each supplier constructs its offered bid above its cost to maximize a profit. In each experiment, one supplier applies the new mechanism and the other suppliers are using the optimal bidding strategy to construct their offered bids. The result compares the number of winning auctions and total profit between using the new mechanism and using the optimal bidding strategy.

CHAPTER 1: GENREAL INTRODUCTION

1.1 Introduction

The U.S. electric power industry has experienced many changes in the last ten years due to the deregulation. The \$220 billion industry, which has been called the last great government-sanctioned monopoly, is slowly being deregulated and opened to competition. In the United States, reforms are being adopted most rapidly in California and the Northeast, but many states are trying to introduce completion and reform regulation. There are three major fuels used in power generation; coal, oil and natural gas. Over half of the electricity being generated comes from coal, a domestically abundant resource, and it is used primarily to produce electricity [National Mining Association 2001]. Producing electricity from coal is about half the cost of using other fuels, which helps to keep energy costs affordable for American families and businesses. Modern technology has helped to eliminate up to 99.5% of pollutants from the burning process, making coal an inexpensive, effective, and clean source of power.

During the 1990's, U.S. coal production continued an established growth pattern, buttressed by steadily increasing demand for coal for electric power generation. At the same time, competing suppliers have been cutting coal prices while delivering a higher quality product. Coal-fueled power plants produce 57% of the U.S.'s electricity. The U.S. coal and electric power industries are tightly coupled: more than 87 percent of total domestic coal consumption is used for generation by utilities, and coal accounts for more than 57 percent of utility power generation. Thus, competitive electricity generation markets will have far-reaching implications for the coal industry. The traditionally stable coal market may absorb some of the volatility of electricity markets. Coal remains the cheapest source of power on earth, compared to natural gas, oil, and even nuclear energy. An inexpensive energy source means a more competitive U.S. economy and lower energy prices for consumers and people on fixed and limited incomes.

In the electric power industry, the emerging electricity market behaves more like an oligopoly where the market is dominated by a few suppliers of large firms than the perfectly competitive market where no supplier or buyer has the power to influence prices in the

market due to a special features such as, a limited numbers of producers, large investment size, transmission constraints, and transmission losses when discourages purchase from distant suppliers.

The formation of electricity markets in California and in many parts of the world is based on an auction mechanism. The sealed bid auction has been widely used in the electricity market format such as in California. Suppliers submit only one sealed bid in each auction where each auction is limited by a period of time or duration. Dispatch orders and prices are determined using the system marginal price (SMP). In most cases, the market price is assumed to be the system marginal price, the price bid for last Megawatts/hour of power purchased to meet system demand, and paid to all the accepted suppliers. The monthly cost of coal purchased from seven different regions in the U.S. is shown in Figure 1. Costs are monthly average costs (cents/Million Btu) received from the U.S. Department of Energy [Energy Information Administration Office of Coal, Nuclear, Electric and Alternate fuels 1981-1990]. The seven regions include New England, Middle Atlantic, EastNorth Central, South Atlantic, EastSouth Central, WestSouth Central, and Pacific.

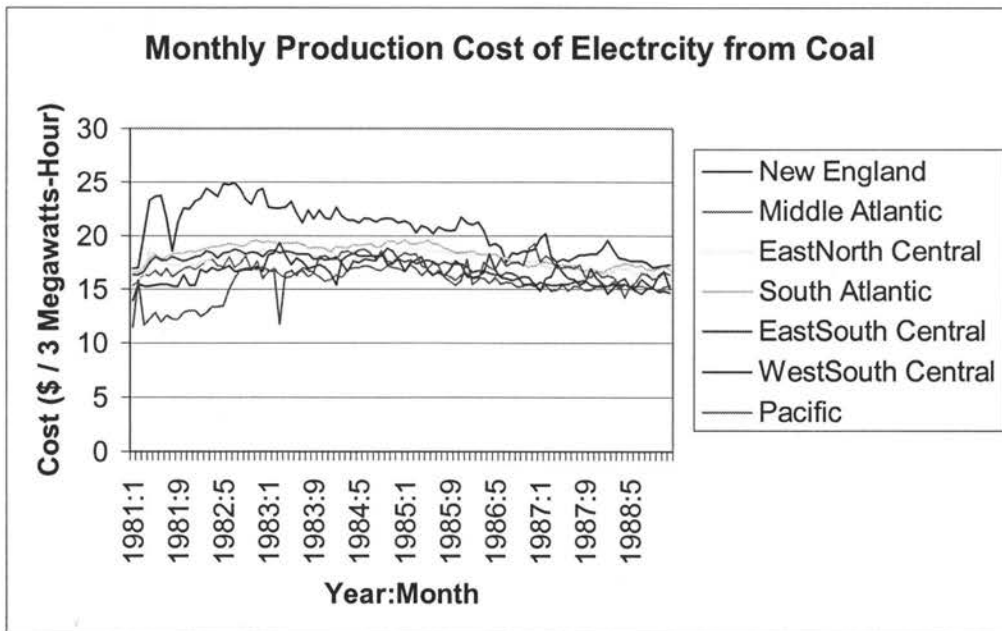


Figure 1: monthly cost of coal purchased from seven regions between January, 1981 and December, 1988

The x-axis in a Figure 1 represents the time span between January, 1981 and December, 1988. The symbols on the x-axis are represented as Year:Month. For example, 1981:1 means January, 1981. The y-axis represents the cost of coal purchased to generate 3 Megawatts electricity per hour. As seen in Figure 1, New England region's cost of coal purchased was fluctuated during the first five years of this information as well as Pacific's being fluctuated between 1981 and 1984. The other regions' costs of coal between January, 1981 and December, 1988 seemed to fluctuate between \$15 and \$19 per 3 Megawatts-Hour.

1.2 Objective

The primary objective of this work is to investigate a mechanism for increasing a supplier's short-term profitability and the number of winning auctions in an electric power auction. Short-term refers to a time period of 24 hours to one week. The new mechanism to adjust the supplier's offered bid to win on a margin, the highest winning offered bid, is proposed and investigated. Assume that all winning suppliers are paid at the system marginal price.

Based on the work of the optimal bidding strategy [Hao 2000], this research will investigate the new mechanism to adjust the offered bid to win on the margin under the clearing price rule. The clearing pricing rule means that all winning suppliers are paid at the highest winning offered bid. This is the price that will be paid to all suppliers by buyers who purchase power from the market. This new mechanism will improve the supplier's chance to win and receive more profit compared to applying the optimal bidding strategy alone in an electric power auction when demand is assumed to know.

Assume that all suppliers have the same capacity to produce the electric power from coal. It is assumed that all suppliers produce the same amount of 3 Megawatts-Hour of electricity in this study. The first step is to construct forecasting models of competitors' production cost from their historical information by employing the Box-Jenkins model concept for a short-term time series forecast. The next step is to employ the optimal bidding strategy to those competitors' production cost in order to determine their optimal bids.

Experiments are conducted using the coal cost data presented in Figure 1. The investigation will compare the total profit and number of winning auctions for suppliers using the new mechanism versus the optimal bidding strategy in 24 auctions. Assume that a supplier profits when its offered bid wins and the bid is above its production cost.

It is assumed that each supplier submits one sealed bid for one block of 3 Megawatts-Hour in each auction. Standard blocks are assumed for the auction, so that bid quantity will not be a factor. The number of winners depends on the number of demands (one demand is one block). Information on the number of participating suppliers, number of

buyers (demand of blocks) and an interval for production cost are usually assumed to be known for all suppliers. The question to be addressed is – will forecasts of competitors' production costs and the new mechanism help a supplier increase its short-term profit and number of winning bids?

CHAPTER 2: LITERATURE REVIEW

For nearly half a century, the U.S. utility companies operated as regulated monopolies characterized by controlled prices. The electric utility industry was considered as natural monopolies marked by economies in scale and size of output making competition wasteful. The chapter begins by reviewing brief history of the U.S. electric utility section then followed by auction systems section. The next section is applying a concept of a short-term times series methods for forecasting electricity and, finally, applying the optimal bidding strategy for the suppliers in the electric power auction.

2.1 Development in the Policy Context

The regulatory policies for utility industry can be summarized by the following acts and regulations. The history of regulation in the U.S. electric industry began when the Federal Power Act of 1935 [Congressional Record 1995] conferred regulatory authority for wholesale, interstate energy transactions to the Federal Power Commission (FPC). FPC was the precursor to the Federal Energy Regulatory Commission (FERC) created in 1977. The Public Utility Holding Company Act (PUHCA) of 1935 was passed to give the Securities and Exchange Commission (SEC) the authority to break up utility holding companies for the malpractice of excessive charges.

The Clean Air Act of 1970 conferred the responsibilities of monitoring the environmental standards to the Environmental Protection Agency (EPA). The majority of electric power generation dependent on fossil fuel was affected by this regulation. The Public Utility Regulatory Policy Act (PURPA) of 1978 [Abel 1992] was one of the important early developments to restructure the electric industry. PURPA consisted of seven titles of which Title I, II, and IV were related to the utility industry. Those titles described the retail regulatory policies for electric utility, competition in electric utility industry, and focused on small hydroelectric power projects.

In 1988, the FERC proposed changes to regulations [U.S. Congress 1988] to promote competition in bidding and independent power production. The U.S. Senate passed a comprehensive National Energy Policy Act (NEPA) in 1992 [Congressional Research

Service 1993] to facilitate the growth of free market electricity. The act consisted of thirty titles of which Title I was the most significant. It focused on energy efficiency issues: reducing the cost of efficiency improvement for generation, transmission, and distribution facilities. In March 1995, FERC proposed to deregulate the wholesale power market [Federal Energy Regulatory commission 1995] by instituting new rules on open access transmission. This ruling meant changes in the sale of electricity between electric utilities and electricity providers. Invariably, this meant an end to electric utility monopolies.

It appeared that the FERC was ready to move far beyond previous pricing policies for electric power industry. Many state utility commissions have already taken initiatives to restructure the utility industry in accordance with the federal proposal. In 1996, the state legislature of California approved Assembly Bill (AB) of 1990 [Ballance 1996]. The law became effective in January 1998, and began a four-year transition to deregulate elements of the electric utility industry. The goal of AB 1890 was to create a competitive marketplace for electricity that would result in reliable sources of energy and lower prices for consumers. The deregulation legislation created two new public agencies to manage California's energy: the California Independent System Operator (Cal-ISO) and California Power Exchange (CalPX). The ISO is responsible for ensuring the reliable transmission of electricity throughout California. The CalPX is a commodity trading exchange that buys and sells energy on the open market for California electric utilities

2.2 Auction as a Market Institution

An auction market can be considered as a trading institution where buyers and sellers can readily meet to maximize their trade gains. McAfee and McMilan (1987) defined auction as "a market institution with an explicit set of rules determining resource allocation and prices on the basis of bids from market participations." In standard auction institutions such as Chicago Board of Trade (CBOT) and New York Mercantile Exchange (NYMEX), all the trade units are standardized. The only component of the trade unit that varies is the price. The market participants efficiently decide for transactions on the basis of prices only. The auction system is a very efficient way to move from cost-based operation to price-based operation. Post (1994) and Sheblé (1993) presented a detailed study of auction institution. Post and Sheblé described four standard types of auction institution: English auction, Dutch

auction, the first-price sealed-bid auction, and the second-price sealed-bid auction. These auction mechanisms employ different methodologies of trading. The sealed-bid auction is typically used for government procurement contracts.

The English auction is the auction most commonly used for selling goods; the price is successively raised until one bidder remains. The word "auction" is derived from the Latin *augere*, which means, "to increase". The Dutch auction is the reverse of the English auction for which the auctioneer calls an initial high price and then lowers the price until one bidder accepts the current price. The Dutch auction is used, for example, for selling cut flowers in the Netherlands, fish in Israel, and tobacco in Canada. For a first-price sealed-bid auction, potential buyers submit sealed bids and the highest bidder is awarded the item. In a sealed-bid auction, each bidder can submit only one bid. For the first-price sealed bid auctions are used in the auctioning of mineral rights for U.S. government-owned land, sales of artwork, real estate, etc. Under the second-price sealed-bid auction, bidders submit a sealed bid. The highest bidder wins the item but pays a price equal not to his own bid but to the second-highest bid. Both the first-price sealed-bid and the second-price sealed-bid auction maximizes the trade gains of the market participants.

Smith (1974) presented a slight variation on the first-price sealed-bid auction called a discriminative sealed bid auction. In this case, the sale quantity is fixed at a specific amount. Smith also presented a variant of the second-price sealed-bid auction for a homogeneous commodity. This variant is called the competitive sealed-bid auction that was the same as discriminative sealed bid except that all bids were filled at the price of the lowest accepted bid. Of the various auction institutions, a sealed-bid method appears to be operationally and structurally suitable for the deregulated electricity industry.

2.3 Examples of Short-Term Forecasting Concept and Optimal Bidding Strategy in the Power Market

2.3.1 Short-Term Forecasting in the Power Market

One of the characteristics of the electric power production is that power it cannot be conveniently store. Therefore, at every instant of time there should be a sufficient amount of electricity production to meet demand. Load forecasting is an important part of electric power system operations. Short-term forecasts of an hour ahead or a day ahead load are needed for economic scheduling of generating capacity. An ARIMA time series model forecasts the current value by means of a linear combination of previous values.

The time series concept has been used previously in the electric power industry to forecast load, production cost, etc. Zunko and Komprej (1991) applied the Box-Jenkins time series analysis methods for a short-term load forecasting of daily electric power consumption data in Slovenia. They concluded that the Box-Jenkins approach proved to be a very efficient way for forecasting load. Valenzuela and Mazumdar (2000) introduced the statistical analysis of electric power production costs. Monte Carlo simulation was used to study time series analysis of actual load data to estimate a production cost. The estimated production cost was derived from contributions of the demand and the generator availabilities. In the current regulated climate, production-costing models are widely used in the electric power industry by the individual utilities for the purpose of forecasting the cost of electricity production.

2.3.2 Optimal Bidding Strategy in the Power Market

Theoretically, in a perfectly competitive market, suppliers should bid at, or very close to, their marginal production cost to maximize their profit. However, the electricity market is not a perfectly competitive market and suppliers may benefit by bidding higher than their marginal cost. The optimal bidding strategy is a method of determining a bid that maximizes benefit based on a supplier's costs, constraints, and anticipation of rival and market behavior. Lamont and Rajan (1997) proposed a simple sub-optimal bidding strategy for the situation where two buyers are competing for a single block of energy but it cannot be

extended to the general case of multiple suppliers. Visudhiphan and Ilic (1999) proposed a dynamic model of strategic bidding for the situation with three power suppliers by utilizing the historical and current market clearing prices. This model is heuristic in principle, and is not directly applicable to the general case with more than three suppliers. Shangyou Hao (2000) proposed a bidding behavior model of suppliers in electricity auction markets under clearing pricing rule and with some simplified bidding assumptions. His proposed strategy will be presented in detail in the next chapter and applied in the model chapter.

CHAPTER 3: METHOD OF ANALYSIS

The chapter has four sections. The first section is an overview of the new mechanism to adjust the supplier's offered bid to win on the margin in the electric power auction. Suppose that there are n suppliers and m demands and the number of suppliers is greater than the number of demands. Let $P = \{p_1, p_2, \dots, p_{n-1}\}$ be the set of competitors' forecast production costs. The next step is to sort P ascending order. Let b_{m-1} and b_m be the optimal offered bid having forecasted production costs p_{m-1} and p_m , respectively, using the optimal bidding strategy. For each auction, b_m is the winner with the highest offered bid and b_{m-1} is the second highest offered bid.

The next step is to apply the optimal bidding strategy with p_{m-1} and p_m from the previous step in order to determine the optimal offered bids, b_{m-1} and b_m where $b_{m-1} \leq b_m$. The last step is to find the average offered bids between b_{m-1} and b_m . This mechanism uses the average of b_{m-1} and b_m in order to maximize the probability of bidding between b_{m-1} and b_m . It is also assumed that the average bid is the mean or expectation between b_{m-1} and b_m . This is based on the assumption that b_{m-1} and b_m are random variables with normal distributions and the same standard deviation.

The next section is an overview of the short-term forecast ARIMA model. A three-stage procedure is followed to find a good model to forecast each competitor's future production cost based on its historical production cost. These three stages are identification (find an ARIMA model), estimation (estimate the parameters of the ARIMA model from the identification's stage), and diagnostic (check the model for adequacy) (Pankratz 1983). After completing the three-stage procedure, the forecast will be performed.

The third section is an overview of the optimal bidding strategy. This study assumes that every supplier follows this strategy and no supplier prefers to change the strategy. The reason is because it would simplify the result of the new mechanisms when all suppliers are employing the same strategy. The optimal bidding strategy was modeled based on bidding behaviors of suppliers in electric auction markets under the clearing pricing rule. The

clearing pricing rule means that all winning suppliers are paid at the highest winning offered bid. This strategy assumed that each supplier's offered bid is above its production cost. The difference between the production cost and the optimal offered bid is called the markup. The markup is calculated from the probability to win on and below the margin. The margin bid is assumed to be the highest winning bid in the auction. Those probabilities are determined from all suppliers' distributed production cost range, the number of demands, and the number of suppliers participating.

The last section is an overview of how to apply the optimal bidding strategy with the new mechanism. There are three experiments in this study and each experiment has four cases. Those four cases are seven suppliers with two demands, seven suppliers with three demands, seven suppliers with four demands, and seven suppliers with five demands. The production cost range is assumed between \$14 and \$23 per 3 Megawatts-Hour based on the production cost of coal during 1984-1988 in Figure 1. The JavaScript web page is employed in order to calculate the optimal offered bid in each case.

3.1 The new mechanism to adjust the offered bid to win on a margin

In this research, Hao's strategy (2000) is used in conjunction with the new mechanism to adjust the supplier's offered bid to win on the margin in the electric power auction. The goal is to determine if we can achieve more profit in the short-term and a greater probability of winning. Hence, in this study, if one supplier employs a forecast time series model to predict competitors' production cost together with the new mechanism to adjust its bid to win on the margin in an electric power auction and assume that all competitors also employ the optimal bidding strategy, will it have more chance to win and more profit compared with using the optimal bidding strategy alone? Assume that one supplier has estimated historical production costs of competitors based on the cost of coal that was purchased from seven different regions. Also, it is assumed that all competitors employ the optimal bidding strategy in order to make up their optimal bids. The main cost of production is assumed to be from the cost of coal purchased. In Figure 2 below, the procedure of adjusting one supplier's offer bid to win on a margin is presented.

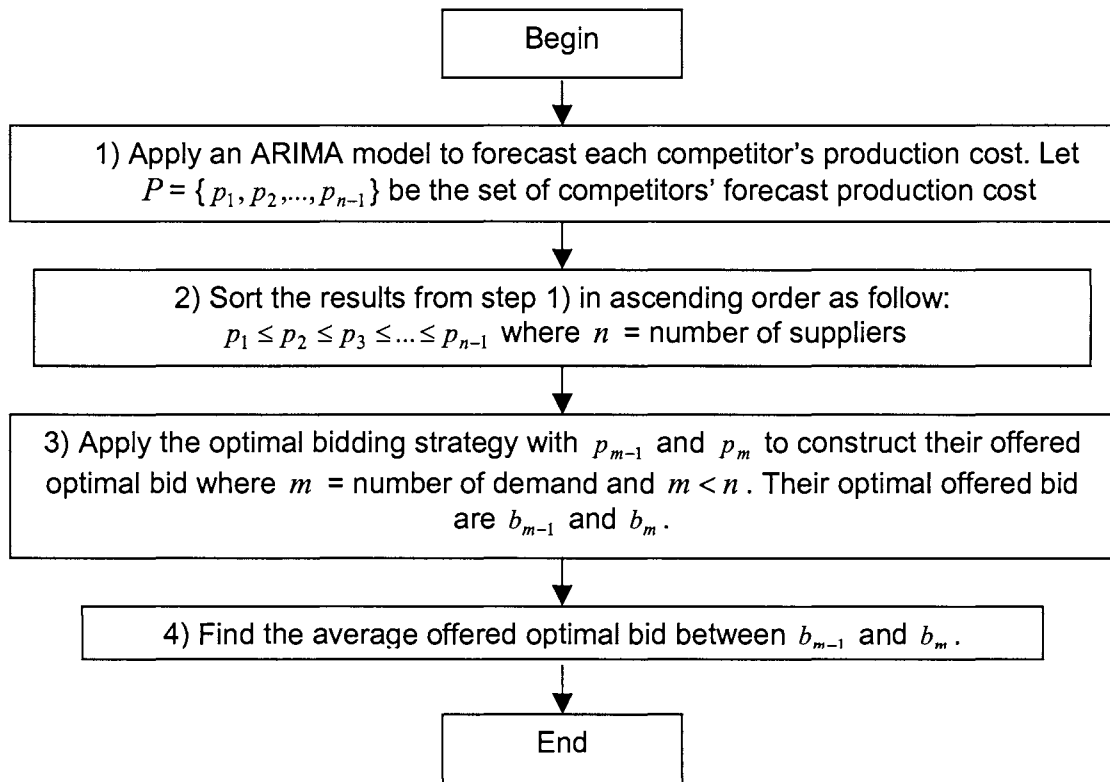


Figure 2: Bid Adjustment Procedure

As seen from Figure 2, the bid adjustment procedure begins by applying an ARIMA model to forecast each competitor's production cost. Each competitor's ARIMA model is constructed based on the monthly historical cost of coal purchased. The historical cost of coal was obtained from the U.S. Department of Energy [Energy Information Administration Office of Coal, Nuclear, Electric and Alternate fuels 1981-1990] for January 1981 and December 1988. It is assumed that the number of suppliers is much greater than the number of demands. Therefore, no supplier has incentive to raise its offered bid so high in order to receive more profit because it will lose the auction when there are less demands compared to many suppliers. To simplify this study, assume that buyers purchase the electricity via the market. The first step is to find a good ARIMA model to forecast each competitor's production cost. The result of this step is ARIMA models for each competitor. Then apply the ARIMA model to forecast the production cost. Let $P = \{p_1, p_2, \dots, p_{n-1}\}$ be the set of competitors' forecast production costs.

The second step is to sort the forecast production costs in ascending order from the previous step and assign them as follows: $p_1 \leq p_2 \leq p_3 \leq \dots \leq p_{n-1}$ where n = number of suppliers. For example, if there are six suppliers (the supplier itself and five competitors) and those competitors' forecast production costs are \$16, \$14.5, \$13, \$15.5, and \$14 per 3 Megawatts-Hour, then the result of ascending sort is $\$13 \leq \$14 \leq \$14.5 \leq \$15.5 \leq \$16$ per 3 Megawatts-Hour, respectively. Then assigning \$13 as p_1 , \$14 as p_2 , \$14.5 as p_3 , \$15.5 as p_4 , and \$16 as p_5 . The third step is to pick p_{m-1} and p_m to construct their offered optimal bids based on the optimal bidding strategy where m = number of demand and $m < n$. The JavaScript program to construct the offered optimal bid based on the optimal bidding strategy is in the last section. The optimal offered bid is calculated from the equation (3.22). The probability of winning but not on the margin is derived from the equation (3.14) and the probability of winning on the margin is from the equation (3.15). Assume that the interval production cost is known.

In fact, the winning bid on the margin in the auction might be different from the expected winning bid on the margin calculated from this mechanism. Hence, p_{m-1} and p_m are chosen to construct the optimal offered bid. For example, if there are three demands, then pick p_2 and p_3 , and apply them with the optimal bidding strategy. After this step, two

optimal offered bids, b_{m-1} and b_m , are determined. The optimal bidding strategy will be described in the third section of this chapter. The last step is to find the average offered optimal bid from the previous step.

The concept of the ARIMA model emphasizes recent past data for the short-term forecast. It would be more effective if the ARIMA model is recalculated each time as a new production cost becomes available. That means repeating the entire cycle of identification, estimation, and diagnostic checking must be repeated. The next production cost for each competitor may be greater or lower than what the ARIMA model of that competitor would forecast. Therefore, the forecast may be not reliable and accurate if we use the same ARIMA model for all the future production cost in this study. This cycle can be repeated quickly with a new production cost because the original model provides a good guide.

3.2 Overview of the short-term forecasting ARIMA model

Box-Jenkins models are often referred to as Auto-Regressive Integrated Moving Average (ARIMA) models. A good overview of the basic theory and modeling procedures can be found in [Pankratz 1983], [Pankratz 1991], and [Enders 1996]. Using the recommended three-stage procedure [Pankratz 1983] shown in Figure 3, an ARIMA model was constructed and evaluated. The three-stage procedure is consisted of Identification stage (find the ARIMA model), Estimation stage (estimate the parameters of the ARIMA model from the identification's stage), and Diagnostic stage (check the model for adequacy).

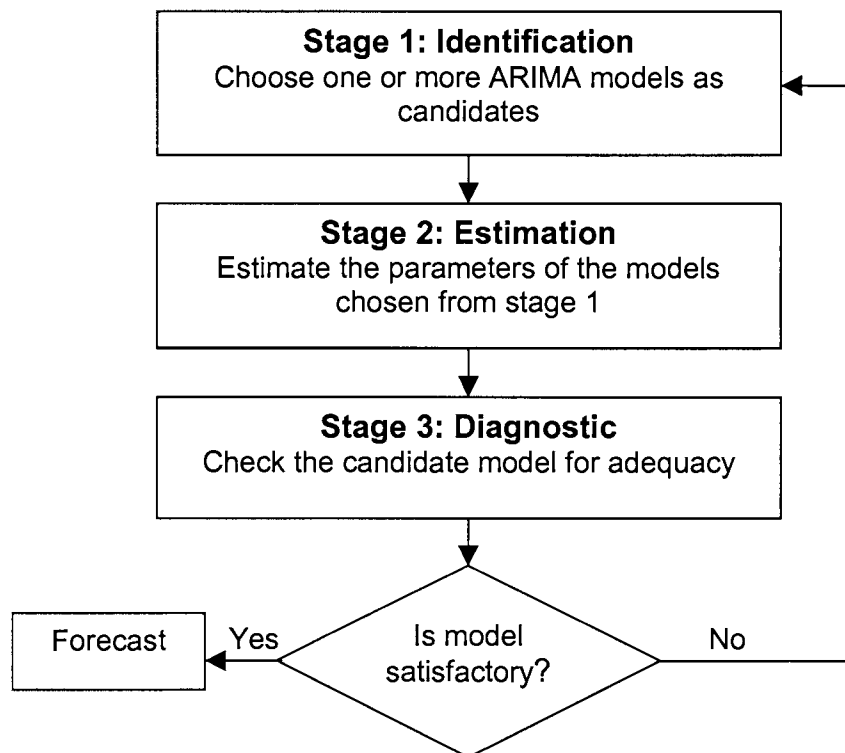


Figure 3: Flowchart of three-stage procedure for finding a good model.

3.2.1 Identification Stage

In the identification stage, stationarity of the series was examined. The stationarity concept helps to get useful estimates of parameters from historical information. Characteristics of stationary time series have a mean (sum of observations and divide by the number of observations), variance (measure the dispersion of the observations around the mean), and autocorrelation function or ACF (measure the statistical relationships between observations in a data series) that are essentially constant through time.

At this stage, autocorrelation function (ACF) and partial autocorrelation function (PACF) are investigated. ACF is used to calculate the autocorrelation coefficient. The estimated autocorrelation coefficient, r_k , is determined from monthly production cost separated by k times periods within a time series. It measures the direction and strength of the statistical relationship between ordered pairs of production costs on two random variables. It is a dimensionless number that can take on values between -1 and $+1$. If $r_k = 0$ then the production cost at time t , z_t , is not correlated to the cost k periods from t , z_{t+k} . A value of -1 means perfect negative correlation and $+1$ means perfect positive correlation.

A decay curve for r_k is an indicator for a series of data that it is stationary. Let r_k be an autocorrelation coefficient of order k , z_t be a production cost at month t , \bar{z} be a mean of this series, n be the number of production costs, and k be a lag length or number of time periods used to calculate each r_k . The r_k is calculated is given by

$$r_k = \frac{\sum_{t=1}^{n-k} (z_t - \bar{z})(z_{t+k} - \bar{z})}{\sum_{t=1}^n (z_t - \bar{z})^2} \quad (3.1)$$

Plotting r_k versus k should show a rapid exponential decay-toward-zero or damp out pattern (suggesting a stationary ACF). It also shows that samples far from each other are

independent. Box and Jenkins (1976) suggest that about 50 observations is the minimum required number of observations to build the ARIMA model.

Based on the result of the ACF, the partial autocorrelation function (PACF) is obtained for a series as below where an estimated partial autocorrelation coefficient, $\hat{\phi}_{kk}$, measures the relationship between \tilde{z}_t and \tilde{z}_{t+k} . Let \tilde{z}_t be $z_t - \bar{z}$ where z_t is a production cost of the data series at time t and \bar{z} is a mean production cost of this series.

$$\hat{\phi}_{11} = r_1$$

$$\hat{\phi}_{kk} = \frac{r_k - \sum_{j=1}^{k-1} \hat{\phi}_{k-1,j} r_{k-j}}{1 - \sum_{j=1}^{k-1} \hat{\phi}_{k-1,j} r_j} \quad (k = 2, 3, \dots) \quad (3.2)$$

$$\text{where, } \hat{\phi}_{kj} = \hat{\phi}_{k-1,j} - \hat{\phi}_{kk} \hat{\phi}_{k-1,k-j}; \quad (k = 3, 4, \dots; j = 1, 2, \dots, k-1)$$

The estimated $\hat{\phi}_{kk}$ is broadly similar to the estimated r_k . The estimated $\hat{\phi}_{kk}$ is also a graphical representation of the statistical relationship between sets of ordered pair $(\tilde{z}_t, \tilde{z}_{t+k})$ drawn from a single time series. Plotting $\hat{\phi}_{kk}$ versus k should show a rapid decay pattern.

However, many production costs are nonstationary (a mean is not constant through time). Therefore, differencing requires the transformation of a nonstationary to stationary. The differencing is a procedure for dealing with a nonstationary mean before choosing ACF and PACF. A series can be differenced once ($d=1$) by calculating the period-to-period changes.

3.2.2 Estimation Stage

Univariate Box-Jenkins (UBJ-ARIMA) models are especially suited to short-term forecasting time-series because they place heavy emphasis on the recent past rather than the distant past [Box and Jenkins 1976]. An ARIMA model is an algebraic statement stating how the production cost (z_t) are related to its past production costs ($z_{t-1}, z_{t-2}, z_{t-3}, \dots$). It deals only with data measured at equally space, discrete time intervals. Time-series data may display a periodic behavior pattern, a pattern that repeats every s time where s represents the length of periodicity and $s > 1$. In this study, $s = 12$ months. UBJ-ARIMA models are also particularly useful for forecasting data series that contain the seasonal observations. With the seasonal observations, the periodic differencing is $z_t - z_{t-s}$. The estimated r_k and $\hat{\phi}_{kk}$ are considered at multiples of lag s ($s, 2s, 3s, \dots$).

At this stage, precise estimates of the coefficients of the model chosen at the Identification stage are investigated by fitting a model to the available data series. Based on the idea of the lag length, which is obtained by an examination of coefficient for autoregressive (AR) terms from ACF and moving average (MA) terms from PACF are estimated by computer software, RATS (Regression Analysis of Time Series) [Enders 1996].

3.2.2.1 ARIMA algebraic form

Since the production cost in this study is monthly ranging over eight years, seasonality is expected. Let P be the maximum lag length on seasonal AR term, Q be the maximum lag length on seasonal MA term, D be the number of seasonal differencing, and s be the length of seasonal differencing (for example, $s=12$ for monthly). The model will be estimated with a seasonal autoregressive (SAR) term, $\Phi_p(B^s)$, and a seasonal moving average (SMA) term, $\Theta_Q(B^s)$ in equation (3.3). Also a seasonal differencing, ∇_s^D , in equation (3.3) is considered as a possibility. A Time series data often display periodic behavior or seasonal that has a pattern which repeats every s time periods where $s > 1$.

Backshift notation is a convenient way of representing ARIMA processes and models. Let p , the lag length of the last PACF spike, be the orders of the AR operator. Let q , the lag length of the last ACF spike, be the order of the MA operator, and d be the number of differencing. Let $\phi_p(B)$ be the p -order AR, $\theta_q(B)$ be the q -order MA, and ∇^d be the d -order differencing.

The backshift operator B^i is used to multiply any time-subscripted variable. The result is that the time subscript is shifted back by i time periods. An ARIMA(p, d, q)(P, D, Q) $_s$ represents that (p, d, q) is the nonseasonal order and (P, D, Q) $_s$ is the seasonal order. The ARIMA(p, d, q)(P, D, Q) $_s$ process can be written in a form of the backshift notation as follows

$$\phi_p(B)\Phi_p(B^s)\nabla^d\nabla_s^D\tilde{z}_t = \Theta_q(B^s)\theta_q(B)a_t \quad (3.3)$$

where,

$$\phi_p(B) = (1 - \phi_1 B - \phi_2 B^2 - \dots - \phi_p B^p) \text{ or the nonseasonal AR operator}$$

$$\theta_q(B) = (1 - \theta_1 B - \theta_2 B^2 - \dots - \theta_q B^q) \text{ or the nonseasonal MA operator}$$

$$\Phi_p(B^s) = (1 - \Phi_s B^s - \Phi_{2s} B^{2s} - \dots - \Phi_{ps} B^{ps}) \text{ or the seasonal AR operator}$$

$$\Theta_q(B^s) = (1 - \Theta_s B^s - \Theta_{2s} B^{2s} - \dots - \Theta_{qs} B^{qs}) \text{ or the seasonal MA operator}$$

$$\nabla^d = (1 - B)^d \text{ or the nonseasonal differencing}$$

$$\nabla_s^D = (1 - B^s)^D \text{ or the seasonal differencing}$$

$$\tilde{z}_t = z_t - \mu$$

3.2.3 Diagnostic Stage

At this stage, some diagnostic checks are required to determine if an estimated model is statistically adequate. The model that fails diagnostic tests is rejected and the procedure will begin at the Identification stage again to find another estimated model. Bartlett (1946) derived an approximate expression for the standard error of r_k . Let n be the number of production costs. This estimated standard error, $s(r_k)$ is calculated as follows:

$$s(r_k) = (1 + 2 \sum_{j=1}^{k-1} r_j^2)^{1/2} n^{-1/2} \quad (3.4)$$

The estimated standard error is used to test the null hypothesis $H_0 : p_K = 0$ for $k = 1, 2, 3, \dots$. The null hypothesis is tested by finding out how far away the sample statistic r_k is from the hypothesized value $p_K = 0$, where how far is a t-statistic equal to a certain number of estimated standard errors. The t-statistic is approximated as follows:

$$t_{r_k} = \frac{r_k - \rho_k}{s(r_k)} \quad (3.5)$$

The result of t_{r_k} implies that if about 5% of the possible r_k falls two or more estimated standard errors away from zero ($p_K = 0$), then the null hypothesis $p_K = 0$ will be rejected since r_k is significantly different from zero at about 5% level. In a similar way, the estimated standard error and t-statistic for $\hat{\phi}_{kk}$ can be tested as follows:

$$s(\hat{\phi}_{kk}) = n^{-1/2} \quad (3.6)$$

$$t_{\hat{\phi}_{kk}} = \frac{\hat{\phi}_{kk} - \phi_{kk}}{s(\hat{\phi}_{kk})} \quad (3.7)$$

where: n = the number of production costs

The null hypothesis is tested by $H_0 : \phi_{kk} = 0$. If the absolute t-statistic of ϕ_{kk} is greater than 2.0 (two or more estimated standard errors), it implies that ϕ_{kk} is different from zero at about 5% significance level and the null hypothesis $\phi_{kk} = 0$ is rejected. The statistically adequate model is the one whose random shocks (a_t) are not autocorrelated. The estimated random shock \hat{a}_t or a residual for any ARIMA model can be calculated by subtracting the calculated value from z_t

$$\hat{a}_t = z_t - \hat{z}_t \quad (3.8)$$

\hat{z}_t is calculated from estimates of parameters rather than known parameters of z_t and depends on $\hat{\mu}$, estimated mean, and the estimated AR and MA coefficients (along with their corresponding past z 's and past residuals.) z_t is an observed value at time t .

Ljung and Box (1978) and Devies (1977) suggest a test statistic based on all the residual autocorrelations as a set. Since it is tedious to check the correlation for all the residuals. If $H_0 : p_K(a) = 0$ indicates that the acceptance of the null hypothesis that no correlations up to lag K exist. Let p_K be the corresponding parameter on all K residual as a set and K be the number of residual autocorrelations jointly tested by the null hypothesis about the correlations among the random shocks and by a Q^* statistics as follows

$$H_0 : p_1(a) = p_2(a) = \dots = p_K(a) = 0 \quad (3.9)$$

with this test statistic

$$Q^* = n(n+2) \sum_{k=1}^K (n-k)^{-1} r_k^2(\hat{a}) \quad (3.10)$$

where

n = number of production costs used to estimate the model

k = number of residual autocorrelations

Q^* has a χ^2 distribution with $(K - m)$ degree of freedom, where m is the number of parameters estimated in the ARIMA model. If Q^* is large (significantly different from zero) at 10% level, then the residual autocorrelation as a set is significantly different from zero and the random shocks of the estimated model are probably autocorrelated. If the calculated of Q^* exceeds the appropriate value in a chi-square table in Appendix B, the null hypothesis of no significant autocorrelations should be rejected. The null hypothesis is found as; H_0 : all residuals up to lag k are not correlated. Rejecting the null hypothesis means accepting an alternative that at least one autocorrelation is not zero.

3.3 Overview of the optimal bidding strategy

The optimal bidding strategy extends the model of traditional first-price auction bidding strategy to supplier's behavior in electricity markets under a clearing-pricing rule. In this scenario, all winning suppliers are paid at the highest winning offered bid. Under a clearing price auction, a supplier winning on the margin, highest winning offered bid, will be paid at the clearing price equal to its bid. To a suppliers winning below the margin, the effect of the clearing price rule is very similar to that of the second price rule [Vickrey 1961], all winning suppliers winning are paid the second highest winning bid. The strategy constructs a bid as a function of a supplier's production cost and the cost distributions of other suppliers. The strategy assumes that all suppliers know the numbers of suppliers and demands as well as the interval production cost.

Let c_i be the production cost of a supplier i and b_i be its bid. Let competitors' production costs be a random variable C drawn from a cost distribution density function $f(C)$ over $[C^1, C^2]$ where C^1 is the lowest and C^2 is the highest production cost, respectively. C is assumed to have a uniform distribution. Let $\Pr\{B(C) > b_i\}$ be the probability that at least one competitor bids more than b_i and $\Pr\{B(C) < b_i\}$ be the probability that at least one competitor bids less than b_i . Based on the uniform distribution function, the probabilities are given by

$$\Pr\{B(C) > b_i\} = \Pr\{C > B^{-1}(b_i) = c_i\} = (C^2 - c_i)/(C^2 - C^1) \text{ and} \quad (3.11)$$

$$\Pr\{B(C) < b_i\} = \Pr\{C < B^{-1}(b_i) = c_i\} = (c_i - C^1)/(C^2 - C^1). \quad (3.12)$$

There are three outcomes for the supplier: winning on the margin, winning below the margin, and losing. Let $R(B^{-1}(b_i))$ be the probability that a supplier who wins the auction, but not on the margin, n be the number of suppliers, and m be the numbers of demands. The probability that a supplier wins but not on the margin is given by

$$R(B^{-1}(b_i)) = \sum_{j=0}^{m-2} \binom{n-1}{j} \Pr\{C < B^{-1}(b_i)\}^j \cdot \Pr\{C > B^{-1}(b_i)\}^{n-1-j}. \quad (3.13)$$

This sums the probabilities found by using a binomial distribution for the number of winners not on the margin. The upper limit is $m - 2$ because there are m winners but two of them, a supplier and one competitor being win on the margin. The other $m - 2$ winners win b_i but not on the margin. Let $H(B^{-1}(b_i))$ in (3.14) be the probability that bidder i exactly wins the auction on the margin. This is the probability that exactly $m - 1$ bidders bid less than b_i and is given by

$$H(B^{-1}(b_i)) = \binom{n-1}{m-1} \Pr\{C < B^{-1}(b_i)\}^{m-1} \cdot \Pr\{C > B^{-1}(b_i)\}^{n-1-(m-1)}. \quad (3.14)$$

However, the common objective for each supplier is to maximize its expected profit. Let the winning bid on the margin be w , the payoff π for the supplier i is $(w - c_i)$ if b_i wins the auction but not on the margin and be $b_i - c_i$ if bid b_i is on the margin. Therefore, the expected payoff function, $\pi(b_i)$, is the sum above of the two terms weighted by their probability of occurrence in (3.15). This can be expressed as,

$$\pi(b_i) = (b_i - c_i)H(B^{-1}(b_i)) + (w - c_i)R(B^{-1}(b_i)). \quad (3.15)$$

Differentiating (3.15) with respect to b_i maximizes the payoff giving

$$\begin{aligned} \frac{d\pi}{db_i} = H(B^{-1}(b_i)) + \left[\frac{dH(B^{-1}(b_i))}{dB^{-1}(b_i)} \frac{dB^{-1}(b_i)}{db_i} (b_i - c_i) \right. \\ \left. + \frac{dR(B^{-1}(b_i))}{dB^{-1}(b_i)} \frac{dB^{-1}(b_i)}{db_i} (w - c_i) \right] = 0. \end{aligned} \quad (3.16)$$

Rewriting (3.16) by applying the formula of inverse function differentiation produces

$$\frac{dB}{dc_i} H + \left[\frac{dH}{dc_i} (B - c_i) + \frac{dR}{dc_i} (w - c_i) \right] = 0. \quad (3.17)$$

Let $\left(\frac{dH}{dc_i}\right)$ be H' , $\left(\frac{dR}{dc_i}\right)$ be R' , and $\left(\frac{dB}{dc_i}\right)$ be B' , then (3.17) can be rearranged as,

$$(BH)' = (H + R)'c - R'w. \quad (3.18)$$

Integrating (3.18) from c to C^2 we obtains (3.19). Because the supplier must auction between c and C^2 in order to obtain a profit. If it wins with the offered bid below its cost, it will not maximize a profit. The boundary condition when $c = C^2$ is $H(B^{-1}(b) = C^2) = 0$ and $R(B^{-1}(b) = C^2) = 0$ because the probability to win on and below the margin with the highest cost C^2 is zero.

$$B(C^2)H(C^2) - B(c)H(c) = \int_c^{C^2} (H + R)'cdc - [R(C^2) - R(c)]w. \quad (3.19)$$

Applying the integration by part formula and boundary condition in (3.19) yields

$$B(c) = c + \frac{\int_c^{C^2} (H(c) + R(c))dc}{H(c)} - \frac{(w-c)R(c)}{H(c)}. \quad (3.20)$$

The equation (3.20) is dependent on the estimated winning bid, w . However, the general bidding strategy in (3.20) is not useful because it depends on the estimated winning bid. If the supplier acts as it was on the margin, the winning bid on the margin is $b = w = B(c)$, then a new function can be defined as follows

$$B(c) = c + \frac{\int_c^{C^2} (H(c) + R(c))dc}{(H(c) + R(c))}. \quad (3.21)$$

From (3.21) above, the optimal bid is based on the supplier's production cost plus the markup. The amount of mark up depends on the probability of winning below and on the

margin that are computed from the cost distribution of all suppliers, market demand, and the number of suppliers participating in the auction.

3.4 Overview of how to apply the optimal bidding strategy in this study

The optimal bidding strategy assumes that the number of suppliers, number of demands and the interval production cost are known. In this study, the production cost range for competitors is between \$14 and \$23 per 3 Megawatts-Hour based on the production cost of coal during 1984-1988 in Figure 1. Competitors use the current production cost for c_i in the next period. As mentioned before, there are four cases in each experiment. After receiving the forecast production costs of competitors and picking p_{m-1} and p_m , those two forecast production costs are applied in the optimal bidding strategy using the same range for C^1 and C^2 (i.e., \$14-\$23). In each case, construct the optimal bid for the production cost p_m first. The equation (3.13) is calculated to find the probability of p_m to win the auction but not on the margin and the equation (3.14) is also calculated to find the probability of p_m to win the auction on the margin. Then apply those two probabilities in the equation (3.21) to derive its optimal offered bid, b_m . The optimal offered bid b_{m-1} can be also calculated from the equation (3.13), (3.14), and (3.21). The JavaScript web-based programs are written for each case in order to demonstrate the probability to win the auction but not on the margin, on the margin, and find the optimal offered bid. The user of this program must fill the following information in the text boxes in order to calculate the optimal offered bid: number of suppliers, number of demands, interval production cost, and its cost of production. For example, Figure 1 in Appendix C is the JavaScript web-based program to calculate the optimal offered bids, b_{m-1} and b_m , when there are seven suppliers and two demands in the first case. After filling those five text boxes and submitting, the program will display the probability to win the auction but not on the margin, the probability to win the auction on the margin, and the combination of those probabilities. If submitting another button located near the left bottom corner, the optimal offered bid will be displayed. The source code for this program is in Table 1 in Appendix C. The JavaScript web-based programs to calculate the optimal offered bid in the second case, the third case, and the fourth case are in Figure 2, Figure 3, and Figure 4 in Appendix C, respectively. Their source codes are also in Table 2, Table 3, and Table 4 in Appendix C.

CHAPTER 4: MODELS AND RESULTS

Due to the confidential information of each supplier's production cost and the prevalent use of coal to generate power, monthly production costs of coal (cents/million Btu) were used from the Energy Information Administration Office of Coal, Nuclear, Electric and Alternate fuels 1981-1990 reports. The monthly production costs of seven different regions (New England, Middle Atlantic, EastNorth Central, South Atlantic, EastSouth Central, WestSouth Central, and Pacific) in the United States are used to present each supplier's monthly production cost to conduct this study. The standard Megawatts-Hours block, 3 Megawatts per hour, is assumed for each auction, so that bid quantity will not be a decision variable. Production cost data for the last 24 months (January, 1989 - December, 1990) was used as future production costs in evaluating the accuracy of the forecasted values by the models. Therefore, monthly costs between 1981 and 1988; the sample size was 96 for each supplier throughout this study.

Cost of production depends on multiple factors such as cost accounting practices, fuel costs, maintenance costs, operation and upgrade costs and, other costs. Given the prevalent use of coal, it is reasonable to use coal costs as an indicator for the production costs. In this research, it is assumed that the majority of the production cost is accounted for by coal costs. Each region is assumed to be one supplier for this study. Distribution costs are not considered. The cost of coal purchased was calculated in dollars per 3 Megawatts-Hour from the average cost (cents/million BTU).

By considering the production cost from the U.S. Department of Energy (1981-1990), this study assumes that each standard block contains 3 Megawatts-Hour and each supplier can bid for only one block for each auction. Based on the optimal bidding strategy, the demand, information on the number of suppliers and cost distribution is assumed known to all suppliers.

It is assumed that all competitors construct their optimal offered bid with the optimal bidding strategy in order to maximize their profit if they win in the sealed bid auction. In addition, no competitor has an incentive to apply any strategies other than that strategy mentioned earlier. All winners are paid at the margin bid. It is also assumed that demand is

constant across all time periods. Production costs of coals are determined by assuming 3,412 Btu of coal can produce 1 Kilowatt-Hour of an electric power [University of Washington, department of Mechanical Engineering 2000].

4.1 Models

There are three experiments in this study. In each experiment, one supplier is being as a supplier applying the new mechanism and the other six suppliers are using the optimal bidding strategy. There are four cases in each experiment (see Table 1 below). In those cases, the number of suppliers is held constant while the number of demands changes. This study selects three suppliers; Middle Atlantic, EastNorth Central, and Pacific, as being the supplier applying the new mechanism and other suppliers use the optimal bidding strategy. Therefore, Middle Atlantic is a supplier and the other six suppliers are competitors in the first experiment. EastNorth Central is a supplier and the other six suppliers are competitors in the second experiment. And Pacific is a supplier and the other six suppliers are competitors in the last experiment.

Table 1: Four cases being conducted in this study

| | Number of suppliers | Number of demands |
|--------------------|----------------------------|--------------------------|
| First case | 7 | 2 |
| Second case | 7 | 3 |
| Third case | 7 | 4 |
| Fourth case | 7 | 5 |

Again, all seven suppliers' production costs are collected from January, 1981 to December, 1990, plotted in ACF and PACF graphs, and modeled in the ARIMA models. The chapter begins with a general modeling using a Box-Jenkins ARIMA model, namely identification, estimation and diagnostics, for all suppliers.

Next, applying the optimal bidding strategy and the a mechanism to conduct a monthly sealed bid auction. A graph for each case shows three bids: the supplier's optimal offered bid, the supplier's average bid, and the margin bid of the winning bid being won on

the margin. The optimal offered bid is the supplier's optimal bid calculated from the optimal bidding strategy. The margin bid is the highest winning bid supposed to win in the auction. And the average bid is the bid calculated from the a mechanism. Finally, the results are evaluated by comparing a total profit and number of winning auctions between applying the new mechanism and applying the optimal bidding strategy alone for 24 auctions. Both average bid and optimal bid can win the auction if they are less than the margin bid.

4.1.1 ARIMA model of New England

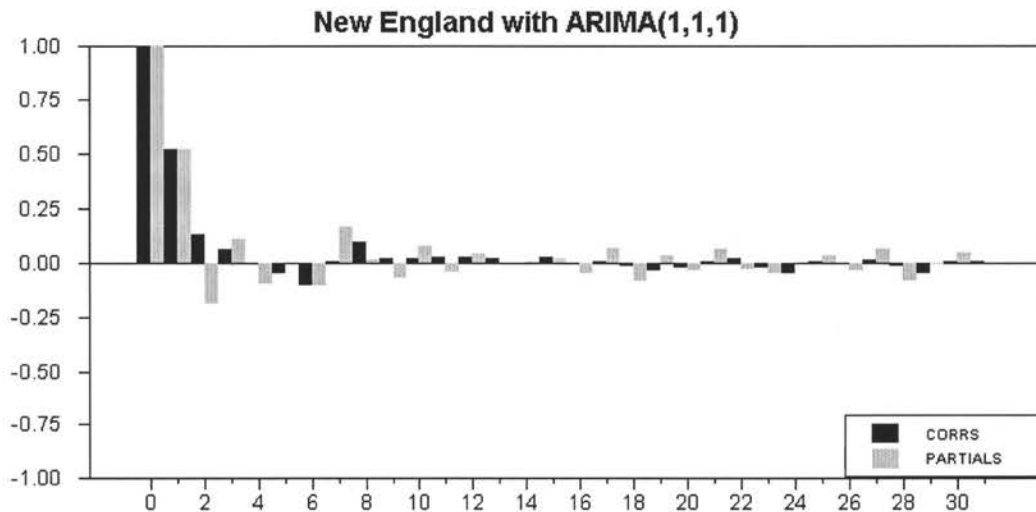


Figure 2: New England with ARIMA(1,1,1)

From New England's production cost in Figure1 in Appendix A and its ACF and PACF graph in the Identification step in figure 2 in Appendix A, its ACF slowly decays to zero indicating that the mean of the production costs is nonstationary. Therefore, differencing is required. The differenced ACF and PACF of New England were presented in Figure 2 above. The shape of differenced ACF and PACF suggests the ARIMA(1,1,1) model with C being constrained to zero because their lags quickly cut off toward zero after lag 1. The ARIMA(1,1,1) implies that the mean of its production cost is not stationary (its production cost appears to be down through time).

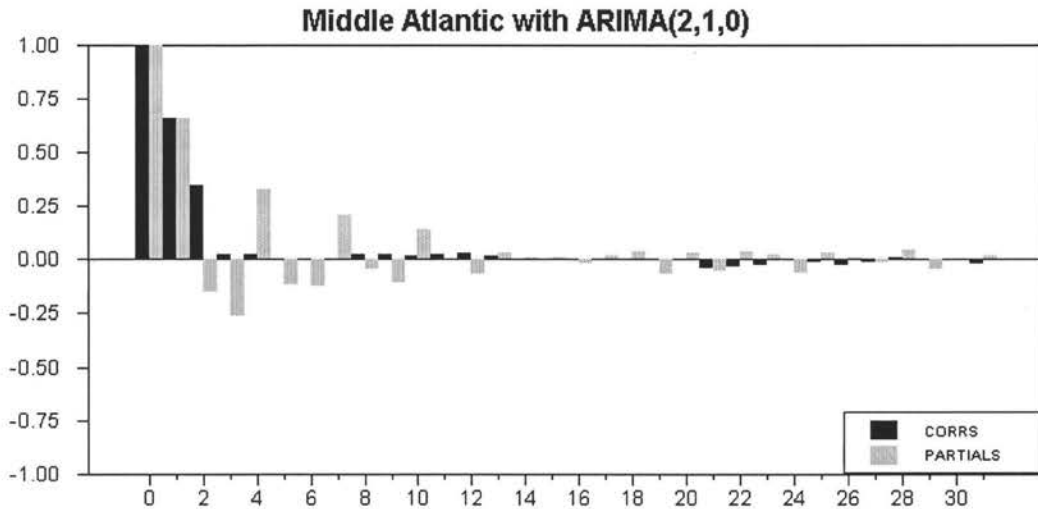
From Table 2 below, all coefficients are statistically significant different from zero roughly at the 5% level because their absolute t-statistic values, in parenthesis after their

coefficients, are more than 2.0. Coefficients may vary each time when recalculating the ARIMA model. Q-statistics are tested for a group of residual autocorrelations calculated at lag 8 and 16, respectively, and they do not exceed the appropriate value in a chi-square table in Table 1 in Appendix B. So the ARIMA(1,1,1) is suitable for New England supplier. Those two coefficients in an equation in Table 2 below are calculated at December 1988 to forecast New England's production cost in January 1989.

Table 2: New England in Estimation and Diagnostic steps

| New England | ARIMA(1,1,1) with C constrained to zero |
|--------------------|--|
| Coefficients | $\phi_1 = -0.57519$ (-3.40439), $\theta_1 = 0.83653$ (7.40147) |
| Backshift notation | $(1 - \phi_1 B)(1 - B)\tilde{z}_t = (1 - \theta_1 B)a_t$ |
| Ljung-Box | Q(8) = 8.3419. Significance Level 0.21411222 |
| Q-Statistics | Q(16) = 15.0347. Significance Level 0.37578356 |

4.1.2 ARIMA model of Middle Atlantic



From Middle Atlantic's production cost in Figure 3 and its ACF and PACF graph in Figure 4 in Appendix A suggest that the mean of its production cost is nonstationary because the ACF slowly decays to zero. Differencing ACF and PACF of Middle Atlantic were presented in Figure 3 above. The shape of differencing ACF and PACF suggests the ARIMA(2,1,0) model with C being constrained to zero. ACF seems to decay while PACF cuts off toward zero. The ARIMA(2,1,0) implies that the mean of production cost from month to month is not constant through time (its production cost appears to be fluctuating).

From Table 3 below, all coefficients are statistically significant different from zero, their absolute t-statistic values, -6.60335 and -2.49236, are more than 2.0. The Q-statistics are tested for a group of autocorrelations calculated at lag 8, 16 and 24, respectively, and all of them do not exceed the appropriate value in a chi-square table in Table 1 in Appendix B.

Table 3: Middle Atlantic in Estimation and Diagnostic steps

| Middle Atlantic | ARIMA(2,1,0) with C constrained to zero |
|--------------------|--|
| Coefficients | $\phi_1 = -0.65584$ (-6.60335), $\phi_2 = -0.24740$ (-2.49236) |
| Backshift notation | $(1 - \phi_1 B - \phi_2 B^2)(1 - B)\tilde{z}_t = a_t$ |
| Ljung-Box | Q(8) = 0.7394. Significance Level 0.99360039 |
| Q-Statistics | Q(16) = 1.4226. Significance Level 0.99999016 |
| | Q(24) = 3.2240. Significance Level 0.99999890 |

4.1.3 ARIMA model of EastNorth Central

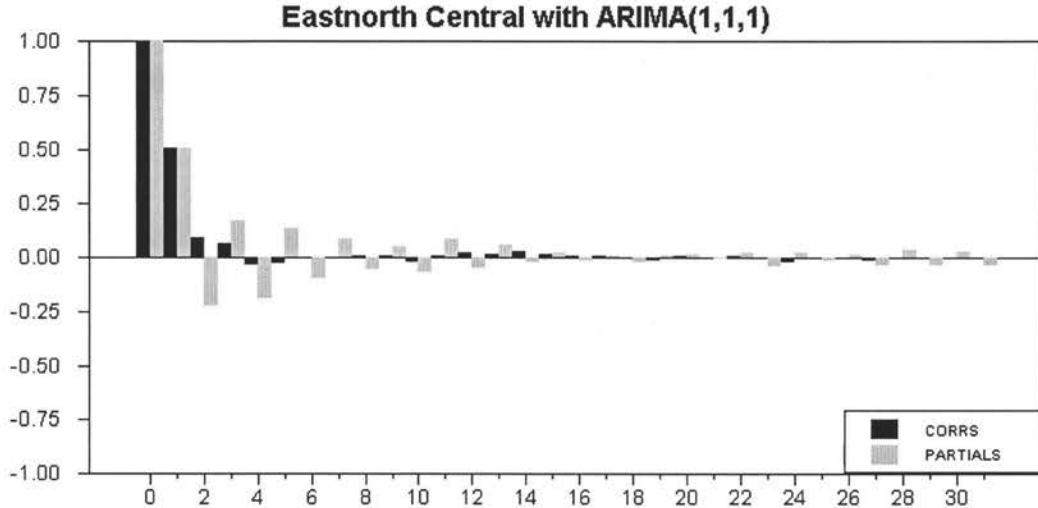


Figure 4: EastNorth Central with ARIMA(1,1,1)

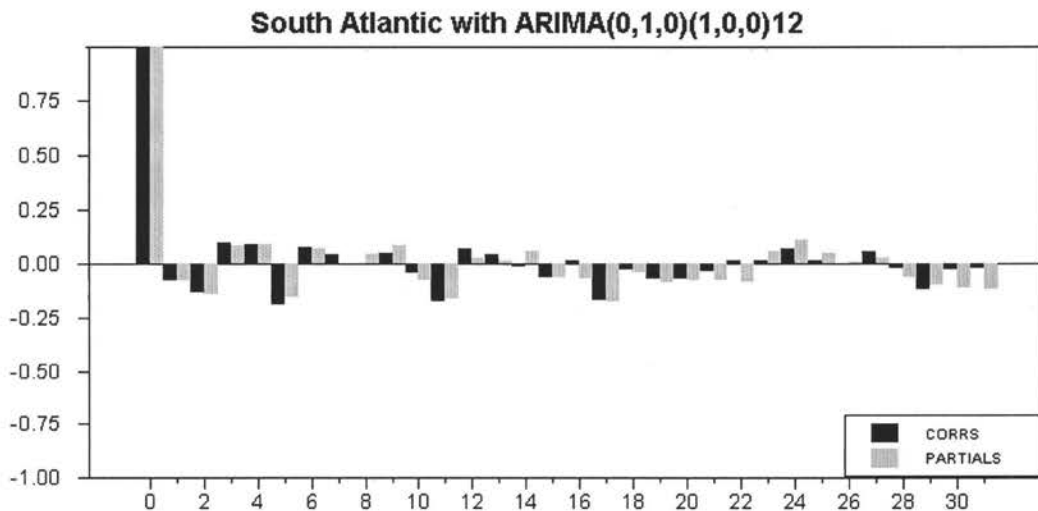
The monthly production cost of EastNorth Central in Figure 5 in Appendix A seems to have a constant mean but its ACF in Figure 6 in Appendix A slowly damps out toward zero. It implies that the mean of the production cost is nonstationary. Therefore, differencing is suggested. Figure 4 above is the differencing ACF and PACF graph. Both ACF and PACF cut off toward zero quickly after lag 1 suggested an ARIMA(1,1,1). ACF is an indicator for a stationary. If it slowly tails off toward zero then its mean is indicated as nonstationary. An ARIMA(1,1,1) of EastNorth Central suggests that the production cost is changing by month to month.

From Table 4, all coefficients are statistically significant different from zero (their absolute t-statistic values are more than 2.0). The Q-statistics at lag 8, 16 do not exceed the appropriate value in a chi-square Table in Table 1 in Appendix B.

Table 4: EastNorth Central in Estimation and Diagnostic steps

| EastNorth Central | ARIMA(1,1,1) with C constrained to zero |
|--------------------|--|
| Coefficients | $\phi_1 = -0.85791$ (-5.07507), $\theta_1 = 0.87589$ (4.96320) |
| Backshift notation | $(1 - \phi_1 B)(1 - B)\tilde{z}_t = (1 - \theta_1 B)a_t$ |
| Ljung-Box | Q(8) = 7.6832. Significance Level 0.26224720 |
| Q-Statistics | Q(16) = 13.1733. Significance Level 0.51292686 |
| | Q(24) = 15.7297. Significance Level 0.82906916 |

4.1.4 ARIMA model of South Atlantic

Figure 5: South Atlantic with ARIMA(0,1,0)(1,0,0)₁₂

The monthly production cost of South Atlantic in Figure 7 in Appendix A is obviously fluctuating; therefore, its mean seems nonstationary. The nonstationary mean is confirmed in Figure 8 in Appendix A when ACF tails off toward zero very slowly. Figure 5 above is the ACF and PACF after being differenced. The first few lags of ACF and PACF graph above are no significantly different from zero, their absolute t-statistic values are less than 2.0, then an ARIMA(0,1,0) is suggested.

If considering a periodic behavior of a monthly production cost time series, 12 months is the length for seasonality. The lags 5, 17, and 29 have a seasonal behavior. The ACF and PACF graph sometimes do not obviously imply if they either tails off or cut off toward zero unless we have to check absolute t-statistic values of their coefficients. Therefore, both seasonal ACF and seasonal PACF are investigated by checking if absolute t-statistic values of their coefficients are significant from zero, in this case, $(1,0,0)_{12}$ is suitable based on its coefficient as well as in Table 5 below.

An ARIMA(0,1,0)(1,0,0)₁₂ is applied for South Atlantic because the mean of its production cost changes over time. The production cost seems to go down after September, 1985. After differencing, the seasonal is provided at lag 5, 17, 29, ... (by length 12).

Table 5: South Atlantic in Estimation and Diagnostic steps

| South Atlantic | ARIMA(0,1,0)(1,0,0) ₁₂ with C constrained to zero |
|--------------------|--|
| Coefficients | $\Phi_{12} = 0.33706$ (3.65523) |
| Backshift notation | $(1 - \Phi_{12}B^{12})(1 - B)\tilde{z}_t = a_t$ |
| Ljung-Box | Q(8) = 7.5580. Significance Level 0.37317267 |
| Q-Statistics | Q(16) = 12.0523. Significance Level 0.67506589 |

4.1.5 ARIMA model of EastSouth Central

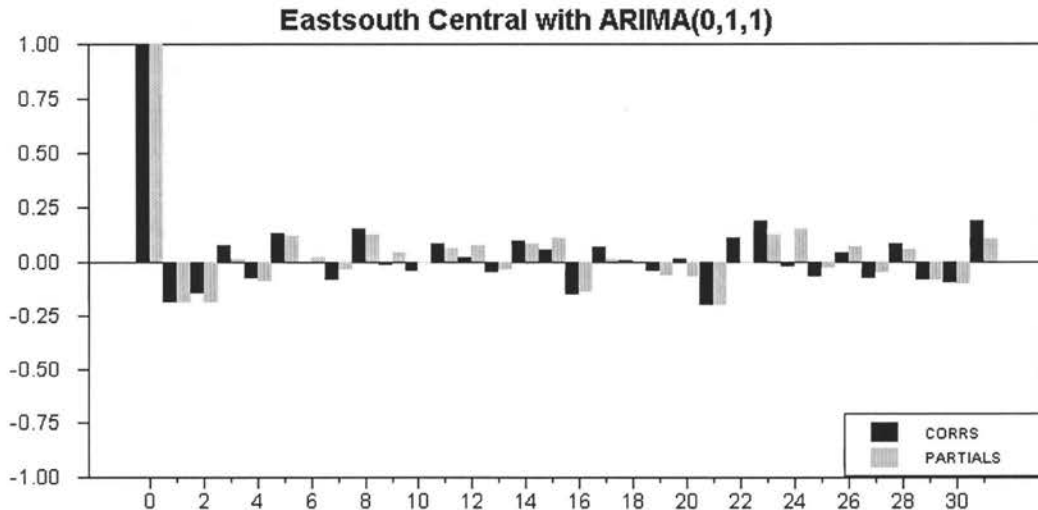


Figure 6: EastSouth Central with ARIMA(0,1,1)

EastSouth Central's production cost in Figure 9 in Appendix A seems to have a stationary mean but its ACF in Figure 10 in Appendix A shows that the mean is nonstationary. Figure 6 above is the ACF and PACF after being differenced. An ARIMA(0,1,1) is suggested because ACF cuts off toward zero after lag 1 while PACF tails off toward zero quickly. It is also suggested that the month-to-month production cost is changing (nonstationary). Its coefficient (θ_1), Backshift notation, and Q-statistic can be seen at Table 6 below.

Table 6: EastSouth Central in Estimation and Diagnostic steps

| EastSouth Central | ARIMA(0,1,1) with C constrained to zero |
|--------------------|--|
| Coefficients | $\theta_1 = -0.25719$ (-2.57508) |
| Backshift notation | $(1 - B)\tilde{z}_t = (1 - \theta_1 B)a_t$ |
| Ljung-Box | Q(8) = 5.7118. Significance Level 0.57377598 |
| Q-Statistics | Q(16) = 10.2999. Significance Level 0.80046239 |

4.1.6 ARIMA model for WestSouth Central

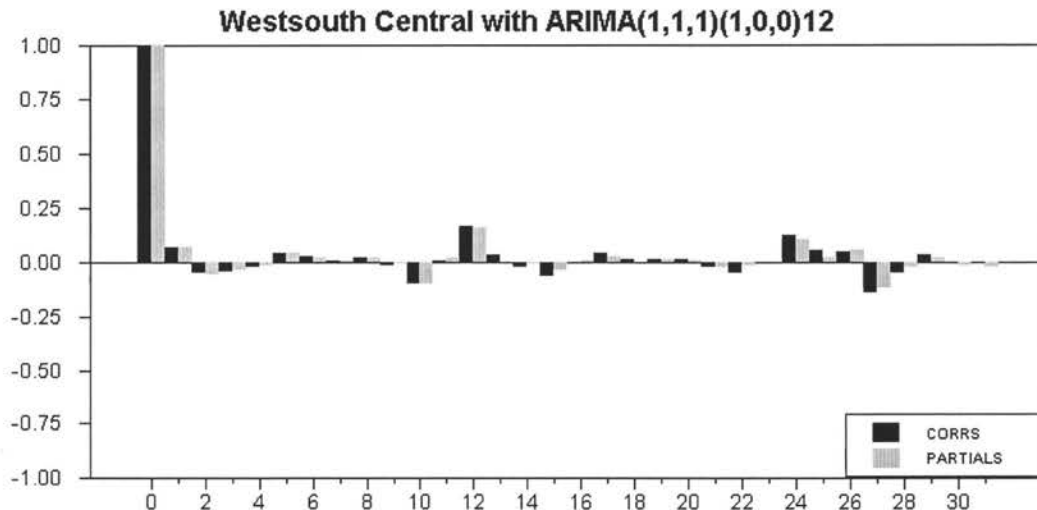


Figure 7: WestSouth Central with ARIMA(1,1,1)(1,0,0)₁₂

Figure 11 in Appendix A presents that WestSouth Central's monthly production cost is periodically changing. Its ACF, in Figure 12 in Appendix A, slowly tails off toward zero that means it has a nonstationary mean. Figure 7 above is the differencing ACF and PACF. An ARIMA(1,1,1) is suitable for WestSouth Central because ACF and PACF cut off toward zero after lag 1 and their absolute t-statistic values are more than 2.0 (Table 7 below).

As mentioned earlier, its production cost is periodically changing over time that implies its mean is nonstationary. The ACF at Figure 7 above contains seasonal observations at lag 12 and 24. The seasonal ACF and PACF suggest an ARIMA(1,1,1)(1,0,0)₁₂ in the identification stage. At the Estimation stage, all coefficients are estimated and their absolute t-statistic values are investigated as well as Q-statistics in Table 7 below.

Table 7: WestSouth Central in Estimation and Diagnostic steps

| | |
|--------------------|---|
| WestSouth Central | ARIMA(1,1,1)(1,0,0) ₁₂ with C constrained to zero |
| Coefficients | $\phi_1 = 0.53716$ (3.99360), $\theta_1 = -0.86862$ (-10.85320), $\Phi_{12} = 0.46728$ (4.52075) |
| Backshift notation | $(1-\phi_1 B)(1-\Phi_{12}B^{12})(1-B)\tilde{z}_t = (1-\theta_1 B)a_t$ |
| Ljung-Box | Q(8) = 0.9570. Significance Level 0.96596084 |
| Q-Statistics | Q(16) = 2.7428. Significance Level 0.99871407 |

4.1.7 ARIMA model for Pacific

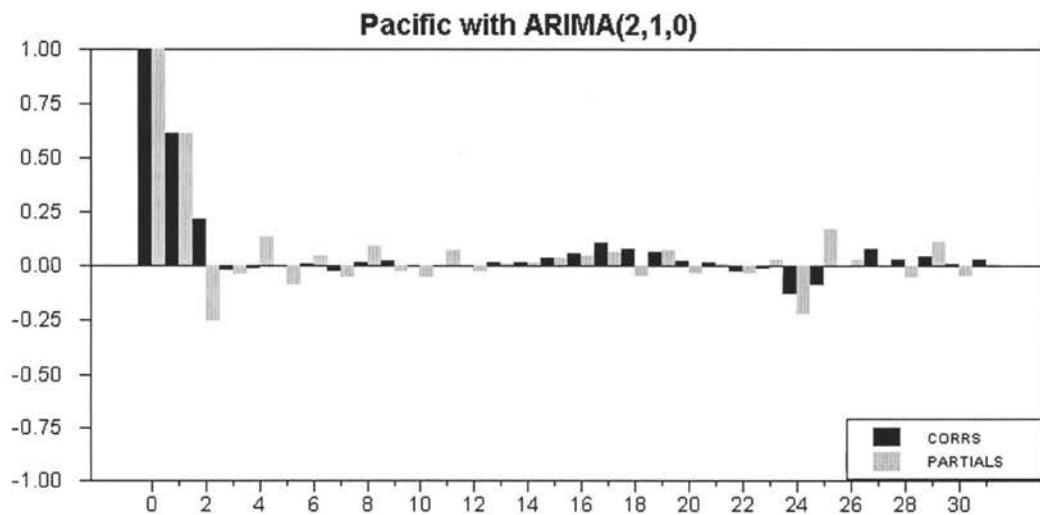


Figure 8: Pacific with ARIMA(2,1,0)

The production costs of Pacific in Figure 13 and ACF and PACF in Figure 14 in Appendix A represent that the mean is not stationary. ACF tails off toward zero slowly. Figure 8 above, the differenced ACF tails off while differenced PACF cuts off toward zero after lag 2 and an ARIMA(2,1,0) is suggested for Pacific. The ARIMA(2,1,0) implies that the mean of the month-to-month production cost is nonstationary and needed to be differenced.

The estimated coefficients in the estimation stage, ϕ_1 and ϕ_2 , are shown in Table 8 below. Their absolute t-statistic values as well as Q-statistics are calculated in the Diagnostic stage.

Table 8: Pacific in Estimation and Diagnostic steps

| Pacific | ARIMA(2,1,0) with C constrained to zero |
|--------------------|--|
| Coefficients | $\phi_1 = -0.40868$ (-4.24921), $\phi_2 = -0.25869$ (-2.91239) |
| Backshift notation | $(1 - \phi_1 B - \phi_2 B^2)(1 - B)\tilde{z}_t = a_t$ |
| Ljung-Box | Q(8) = 6.2956. Significance Level 0.39090015 |
| Q-Statistics | Q(16) = 10.4895. Significance Level 0.72559007 |

4.2 Results

In this section, the number of winning auctions and total profit in 24 auctions will be compared between applying the new mechanism and optimal bidding strategy. There are three cases in this section. First case, Middle Atlantic is the supplier applying the new mechanism to adjust its bid to win on the margin and other suppliers are using the optimal bidding strategy to construct their optimal bid. The second and third cases are assigning EastNorth Central and Pacific as a supplier.

In this experiment there are monthly auctions from January, 1989 to December, 1990 (24 auctions mentioned before). In each auction, assume that they are seven suppliers competing to win to supply an electric power. The experiment compares the result when they are seven suppliers with two demands, three demands, four demands, and five demands. The supplier's average bid is calculated from the new mechanism. The supplier's optimal bid is calculated from the optimal bidding strategy. And the margin bid is also calculated from the optimal bidding strategy, where this bid is supposed to win on the margin (if all competitors, including the supplier itself, are using the optimal bidding strategy in the auction).

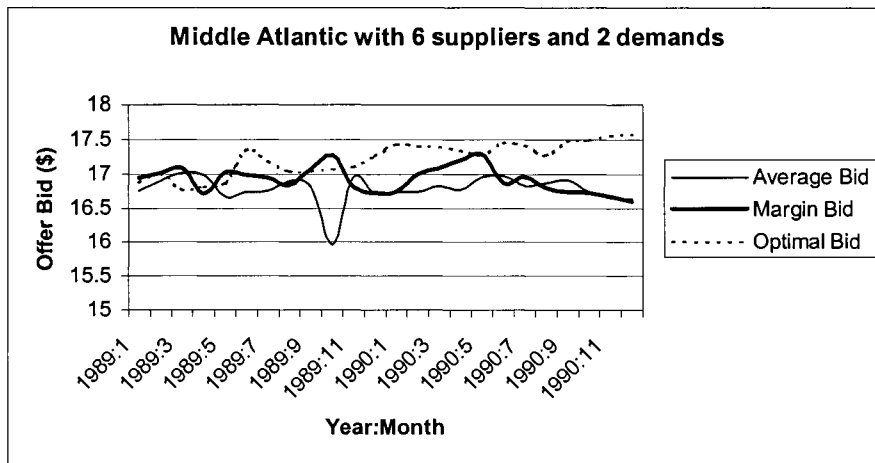
In each graph, the x-axis represents year and month starting from January, 1989, to December, 1990. The y-axis represents the supplier's optimal bid, the average bid obtained from the new mechanism, and the margin bid. The graph presents the supplier's own optimal bid and the average bid together in order to compare their offered bid against the margin bid. The number of winning auctions, percentage of winning auctions, and total profit of applying the new mechanism and applying the optimal bidding strategy in 24 auctions are listed in each graph as well.

In each auction, the average bid is compared with the margin bid in order to provide if the average bid is a winning bid on the margin, it must be smaller than the margin bid, and the profit, subtracting the production cost from the average bid. The profit is zero when the supplier does not win that auction. The optimal bid is also compared with the margin bid in the same way. The total number of winning auctions and total profit are calculated from 24 auctions.

4.2.1 Case 1: Middle Atlantic is a supplier

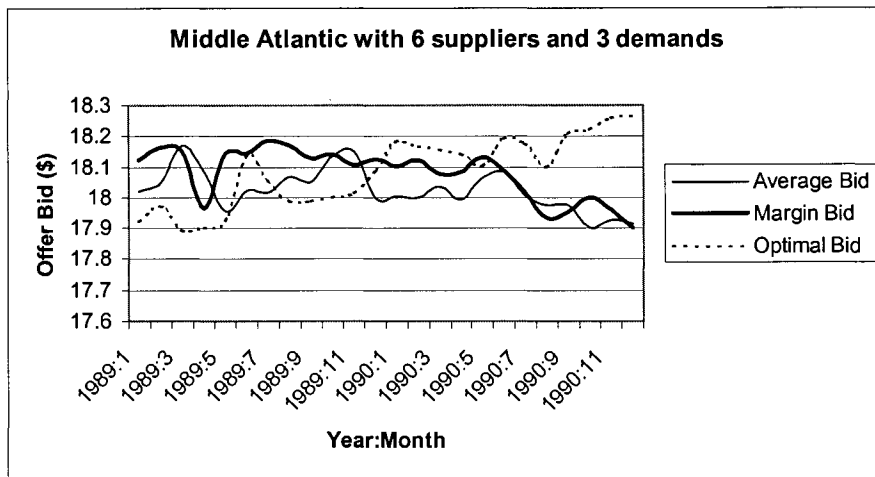
When applying the new mechanism in a 2-demands auction in Figure 9, its percentage of winning auction is 62.50% compared with 37.50% from applying the optimal bidding strategy and its total profit is \$18.235 while \$14.414 with the optimal bidding strategy. At the auction with 3 demands in Figure 10, the new mechanism increases the percentage of winning auction to 75.00% from 58.33% and its total profit also increases to \$43.08.

But at the auction with 4 demands in Figure 11, the number of winning auctions decreases to 62.50% from 70.83%. Its profit is also reduced to \$53.331 from \$63.759. The interesting result happens at the auction with 5 demands, the number of winning auctions from both of them is 100% but their profits are different.



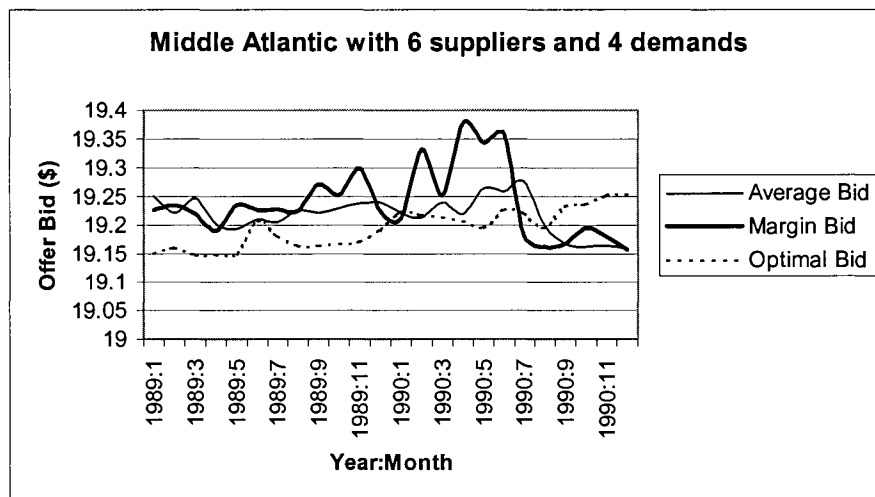
| | Number of winning auctions | Percentage of winning auctions | Total profit (\$) |
|------------------------------|----------------------------|--------------------------------|-------------------|
| Apply with the new mechanism | 15 | 62.50% | \$18.235 |
| The optimal bid | 9 | 37.50% | \$14.414 |

Figure 9: Middle Atlantic with the result in 2 demands auction



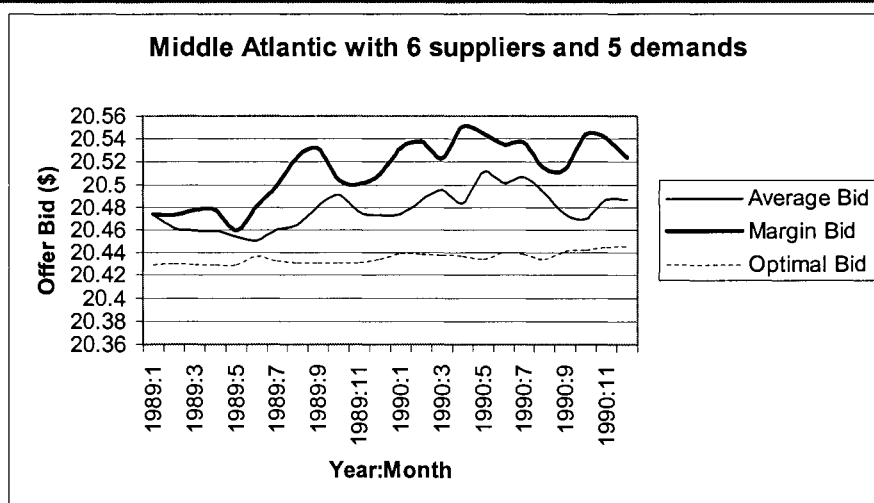
| | Number of winning auctions | Percentage of winning auctions | Total profit (\$) |
|------------------------------|----------------------------|--------------------------------|-------------------|
| Apply with the new mechanism | 18 | 75.00% | \$43.080 |
| The optimal bid | 14 | 58.33% | \$38.619 |

Figure 10: Middle Atlantic with the result in 3 demands auction



| | Number of winning auctions | Percentage of winning auctions | Total profit (\$) |
|------------------------------|----------------------------|--------------------------------|-------------------|
| Apply with the new mechanism | 15 | 62.50% | \$53.331 |
| The optimal bid | 17 | 70.83% | \$63.759 |

Figure 11: Middle Atlantic with the result in 4 demands auction



| | Number of winning auctions | Percentage of winning auctions | Total profit (\$) |
|------------------------------|----------------------------|--------------------------------|-------------------|
| Apply with the new mechanism | 24 | 100.00% | \$117.412 |
| The optimal bid | 24 | 100.00% | \$118.247 |

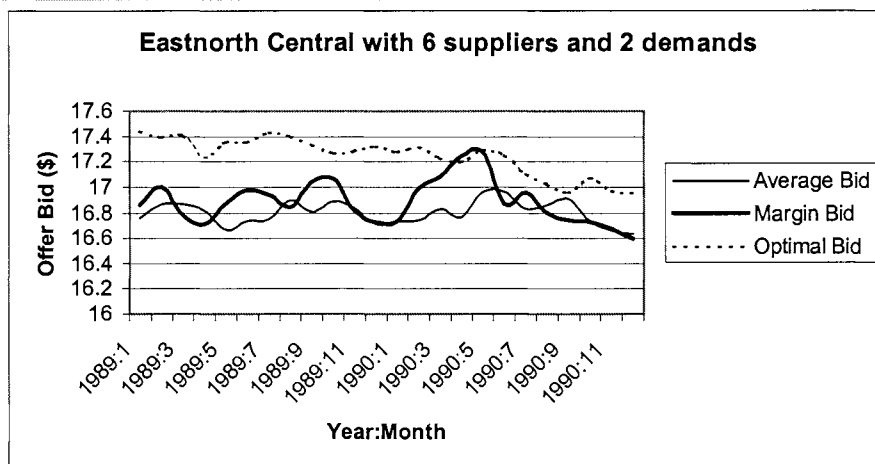
Figure 12: Middle Atlantic with the result in 5 demands auction

4.2.2 Case 2: EastNorth Central is a supplier

When applying the new mechanism in a 2-demands auction in Figure 13, the new mechanism increases the number of winning auctions by 58.33% comparing with 4.17% from applying the optimal bidding strategy and its profit also increases to \$15.156 from \$1.633. Notice that the optimal offered bid line is above the margin bid line almost all the time indicated that its production costs are high; hence, it has not much chance to win on the margin.

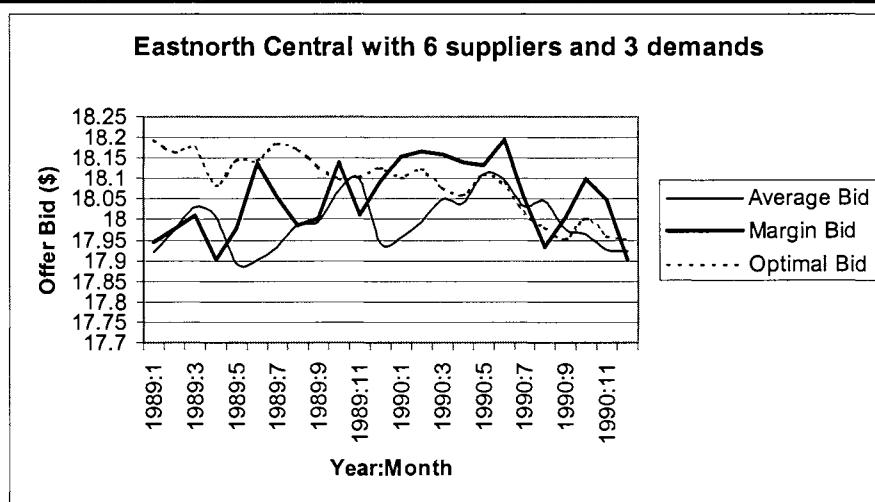
In the 3-demands auction in Figure 14, the new mechanism improves 75.00% chance to win on the margin compared with 50.00%. The new mechanism also increases a profit \$42.126 from \$30.851. However, in the 4-demands auction in Figure 15, the number of winning auctions of both applying the new mechanism and optimal bids is 87.50% but the total profit of applying the new mechanism is lower than the other. The result of the 5-demands auction in Figure 16 is the same way as in a 4-demands auction that the number

of winning for both of them is 100.00% but the total profit of applying the average bids is lower, \$116.309.



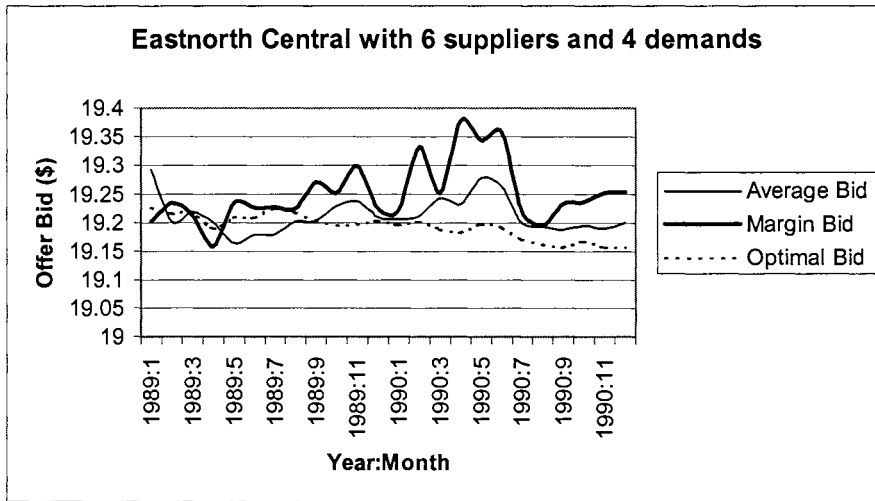
| | Number of winning auctions | Percentage of winning auctions | Total profit (\$) |
|------------------------------|----------------------------|--------------------------------|-------------------|
| Apply with the new mechanism | 14 | 58.33% | \$15.156 |
| The optimal bid | 1 | 4.17% | \$1.633 |

Figure 13: EastNorth Central with the result in 2 demands auction



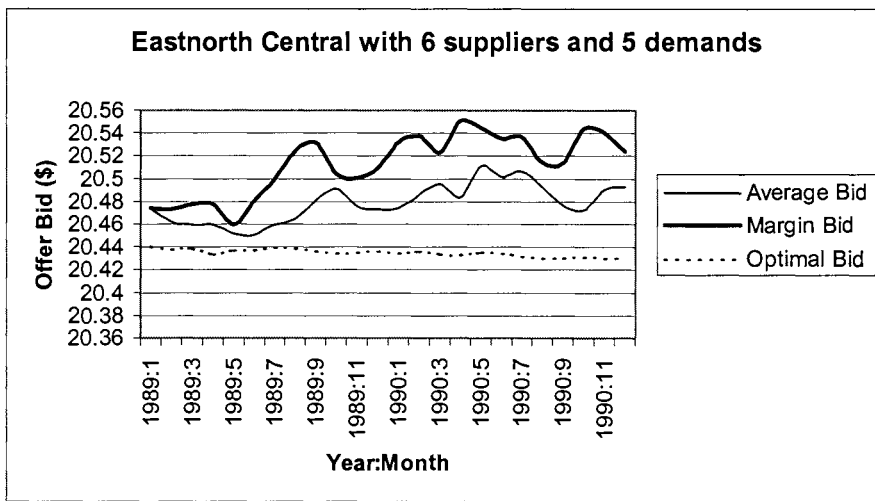
| | Number of winning auctions | Percentage of winning auctions | Total profit (\$) |
|------------------------------|----------------------------|--------------------------------|-------------------|
| Apply with the new mechanism | 18 | 75.00% | \$42.126 |
| The optimal bid | 12 | 50.00% | \$30.851 |

Figure 14: EastNorth Central with the result in 3 demands auction



| | Number of winning auctions | Percentage of winning auctions | Total profit (\$) |
|------------------------------|----------------------------|--------------------------------|-------------------|
| Apply with the new mechanism | 21 | 87.50% | \$75.714 |
| The optimal bid | 21 | 87.50% | \$76.761 |

Figure 15: EastNorth Central with the result in 4 demands auction

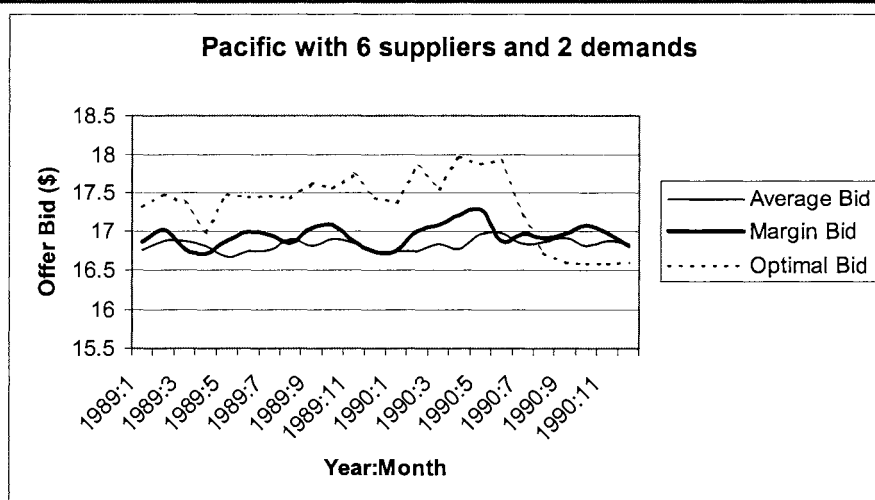


| | Number of winning auctions | Percentage of winning auctions | Total profit (\$) |
|------------------------------|----------------------------|--------------------------------|-------------------|
| Apply with the new mechanism | 24 | 100.00% | \$116.309 |
| The optimal bid | 24 | 100.00% | \$117.131 |

Figure 16: EastNorth Central with the result in 5 demands auction

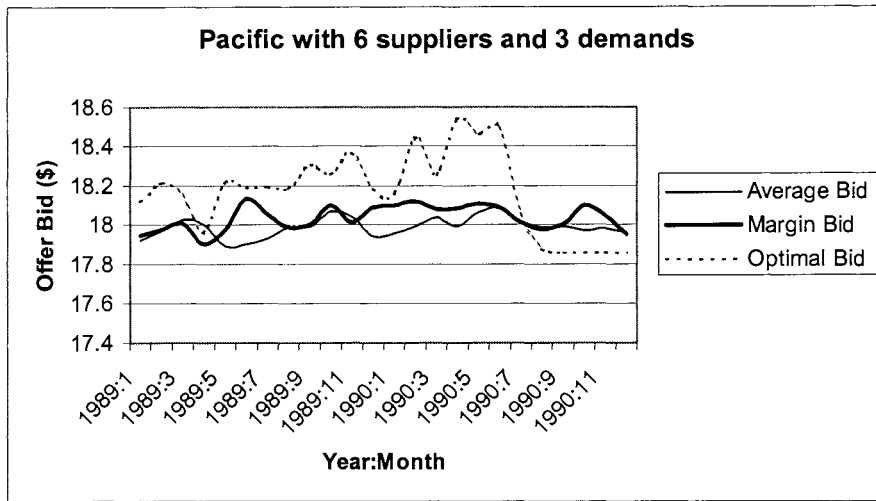
4.2.3 Case 3: Pacific is a supplier

In a 2-demands auction in Figure 17, the percentage of winning auctions is 75.00% with the new mechanism for 24 auctions while it is only 20.83% with the optimal bidding strategy. The profit of applying the new mechanism is \$20.104 while it is only \$13.773 in the optimal bids. The similar result can be seen in the 3 demands auction in Figure 18 where the percentage of winning auctions of applying the new mechanism is higher than that with the optimal bidding strategy, 75.00% against 20.83%, and its profit also higher than the other. In a 3-demands auction, the optimal bids are above the margin bids in 19 auctions while applying the new mechanism are above the margin bid only 6 auctions. Therefore, the total profit of apply the new mechanism is also higher than that in the other one approximately \$19.801 ($\$38.964 - \$19.163 = 19.801$). In a 4-demands auction in Figure 19, the result is also similar. Applying the new mechanism has more chances to win than applying the optimal bidding strategy and its total profit is also higher. Its percentage of winning auctions is 87.50% and its total profit is \$74.336. While, if applying the optimal bidding strategy alone, its percentage of winning auctions is only 41.67% and its total profit is about \$43.157.



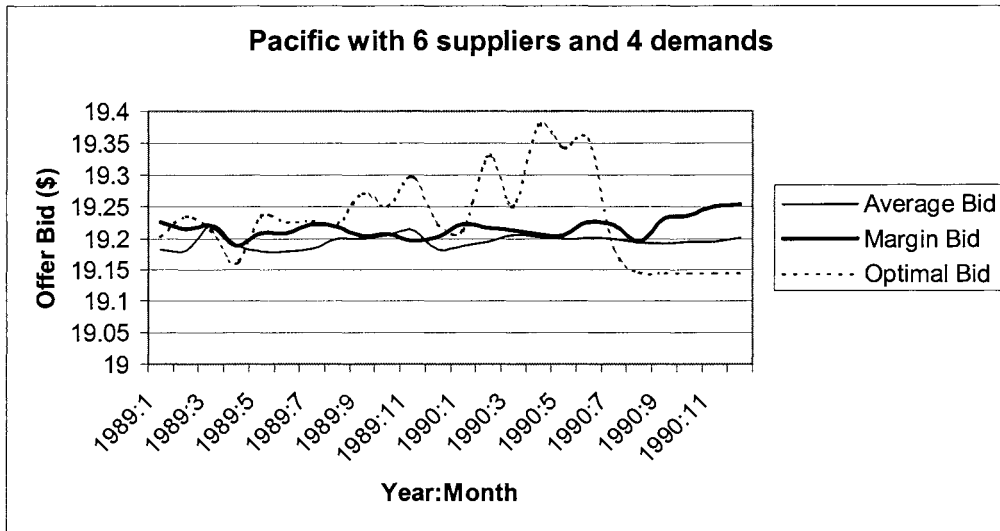
| | Number of winning auctions | Percentage of winning auctions | Total profit (\$) |
|------------------------------|----------------------------|--------------------------------|-------------------|
| Apply with the new mechanism | 18 | 75.00% | \$20.104 |
| The optimal bid | 5 | 20.83% | \$13.773 |

Figure 17: Pacific with the result in 2 demands auction



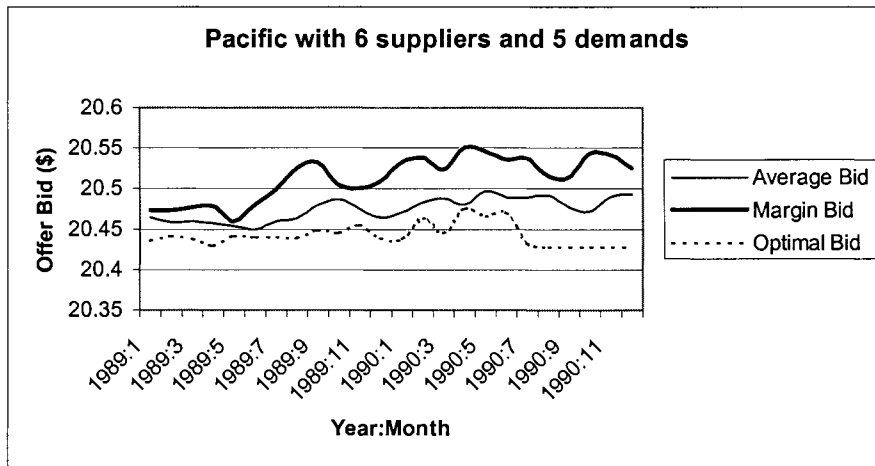
| | Number of winning auctions | Percentage of winning auctions | Total profit (\$) |
|------------------------------|----------------------------|--------------------------------|-------------------|
| Apply with the new mechanism | 18 | 75.00% | \$38.964 |
| The optimal bid | 5 | 20.83% | \$19.163 |

Figure 18: Pacific with the result in 3 demands auction



| | Number of winning auctions | Percentage of winning auctions | Total profit (\$) |
|------------------------------|----------------------------|--------------------------------|-------------------|
| Apply with the new mechanism | 21 | 87.50% | \$74.336 |
| The optimal bid | 10 | 41.67% | \$43.157 |

Figure 19: Pacific with the result in 4 demands auction



| | Number of winning auctions | Percentage of winning auctions | Total profit (\$) |
|------------------------------|----------------------------|--------------------------------|-------------------|
| Apply with the new mechanism | 24 | 100.00% | \$115.270 |
| The optimal bid | 24 | 100.00% | \$116.180 |

Figure 20: Pacific with the result in 5 demands auction

CHAPTER 5: CONCLUSIONS AND DISCUSSION

5.1 Conclusions

A mechanism was presented to adjust one supplier's offered bid to win on the margin in the electric power auction by forecasting competitors' production cost and applying the optimal bidding strategy. Univariate time series analyses are conducted to build an ARIMA model for production costs. The accurate forecast for competitors' production cost depends on how well the ARIMA model fits its previous information as well as the availability of production cost information.

According to 11 successful winning cases from total 12 cases in the result section of chapter 4, a number of winning auctions of applying the new mechanism is higher than or equals to a number of winning auctions of applying the optimal bidding strategy indicated that applying the new mechanism improves a number of winning auctions compared with applying the optimal bidding strategy. If a number of winning auctions of applying the new mechanism are higher than that of applying the optimal bidding strategy, its total profit is also higher.

It appears that the new mechanism outperforms the optimal bidding strategy because of more accurate information on the production costs. The bids are based on the uniform distribution of the production costs. Therefore, a better estimate of the minimum and maximum production costs should offer a short-term advantage.

5.2 Discussion

This work is limited by assuming that all competitors are using the same strategy, the optimal bidding strategy, to construct their optimal offered bid. In fact, each supplier has its own strategy to create its offered bid to win the auction and maximize its profit. An electric power demand varies with time. For example, the demand is high during summer and winter seasons and low during spring season. Therefore, the mechanism should be flexible to adjust a number of demands instead of a constant demand. The production cost of each supplier may be not the same as the production cost from coal in this work because each

supplier might add other costs into its production cost. This work assumes that all suppliers raise their bids above their production cost. But, in fact, some suppliers may auction with an offered bid that is less than its production cost in order to block other suppliers to win the auction.

APPENDIX A: A PRODUCTION COST GRAPH AND THE IDENTIFICATION GRAPH FOR EACH SUPPLIER

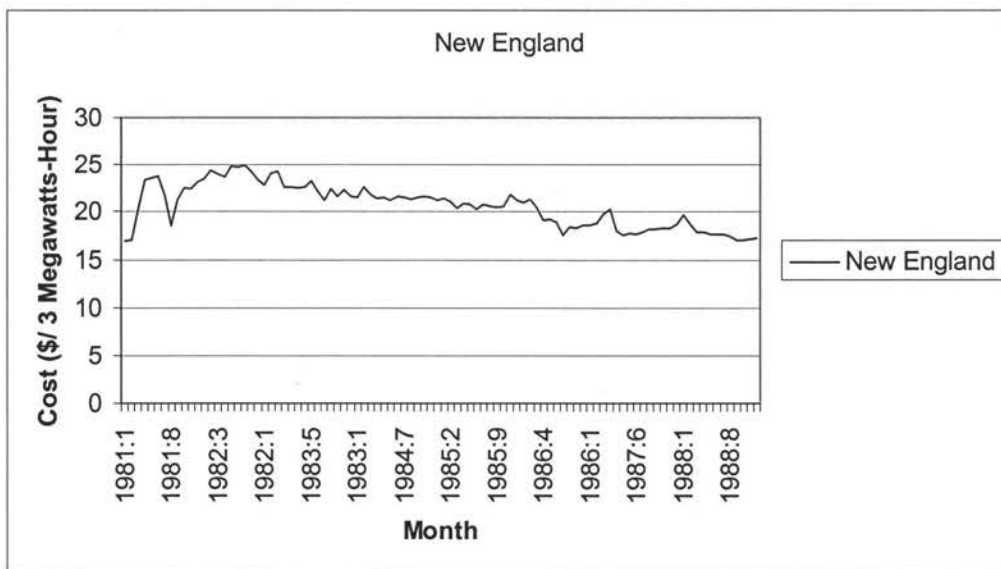


Figure 1: New England's Production Cost

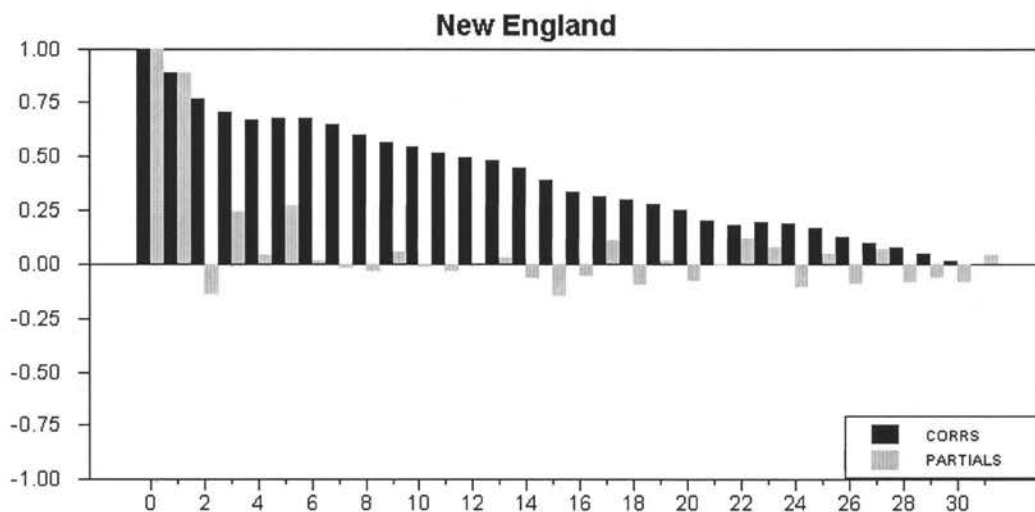


Figure 2: New England in the Identification Step

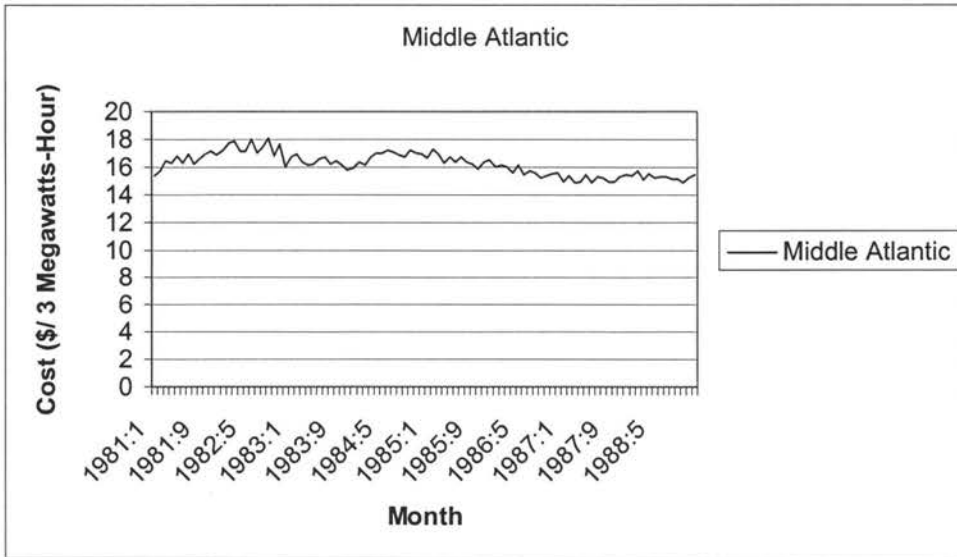


Figure 3: Middle Atlantic's Production Cost

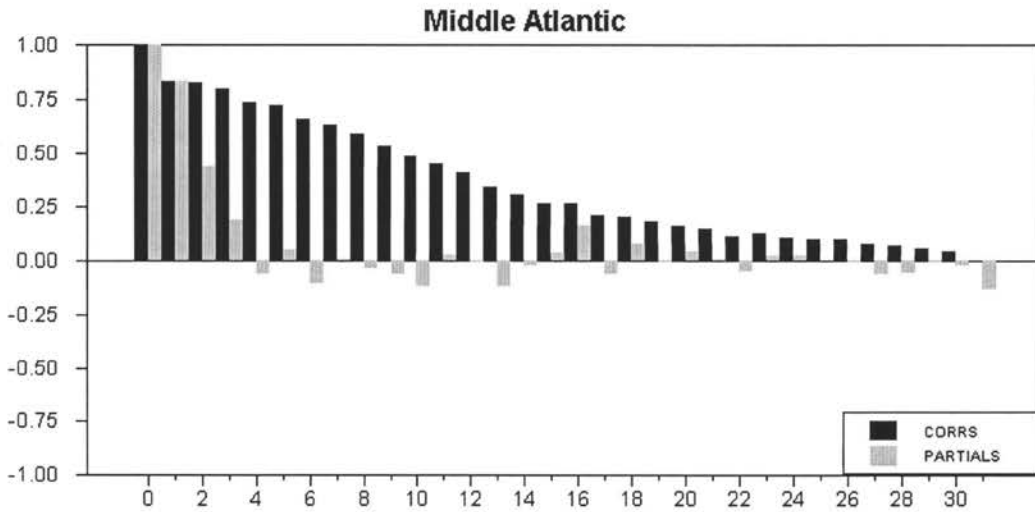


Figure 4: Middle Atlantic in the Identification Step

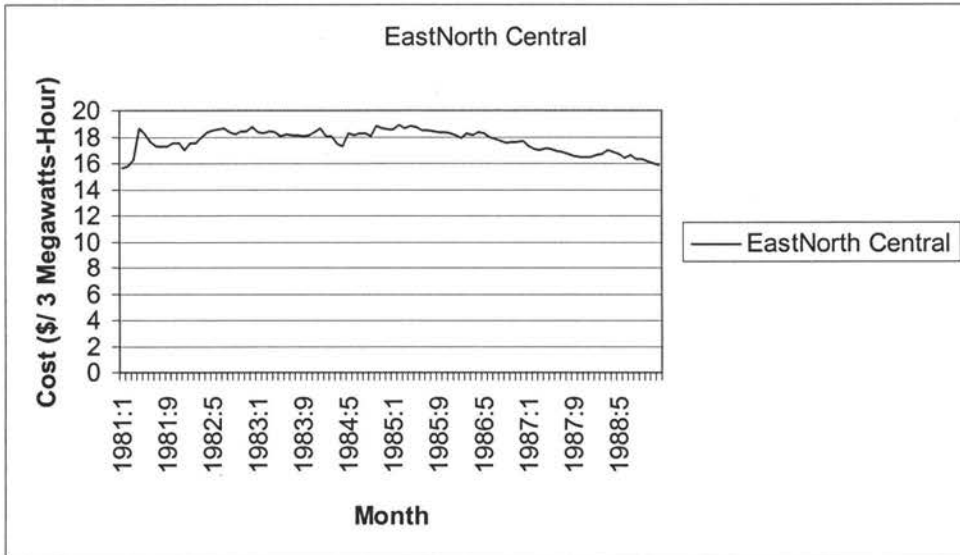


Figure 5: EastNorth Central's Production Cost

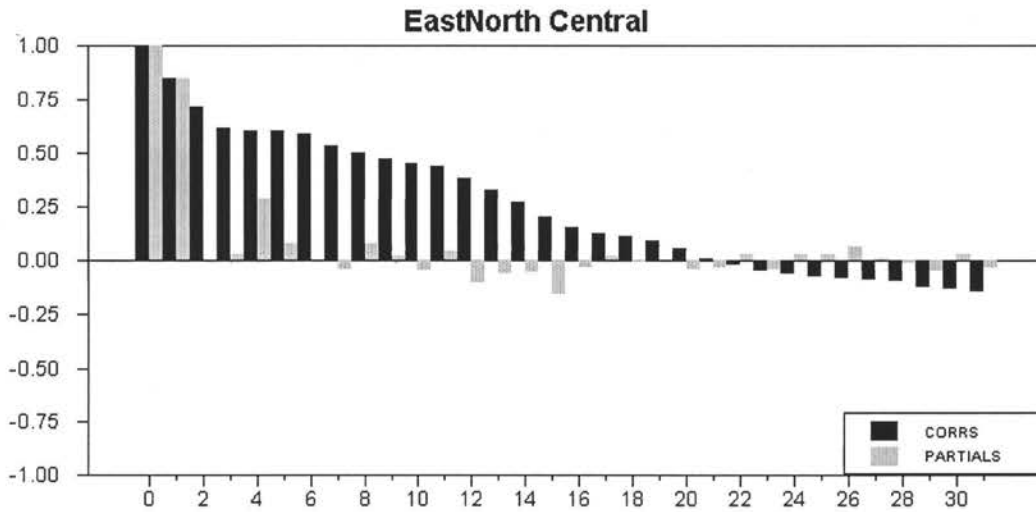


Figure 6: EastNorth Central in the Identification Step

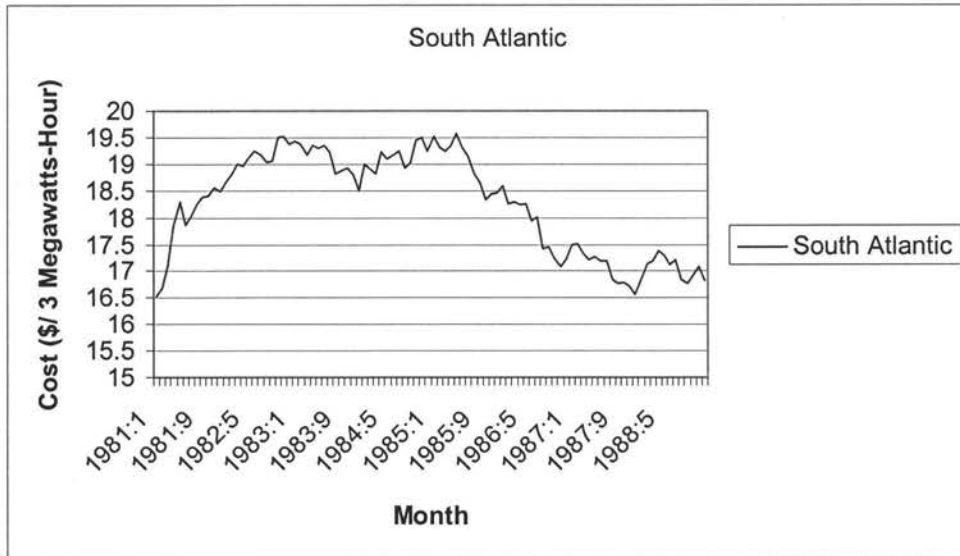


Figure 7: South Atlantic's Production Cost

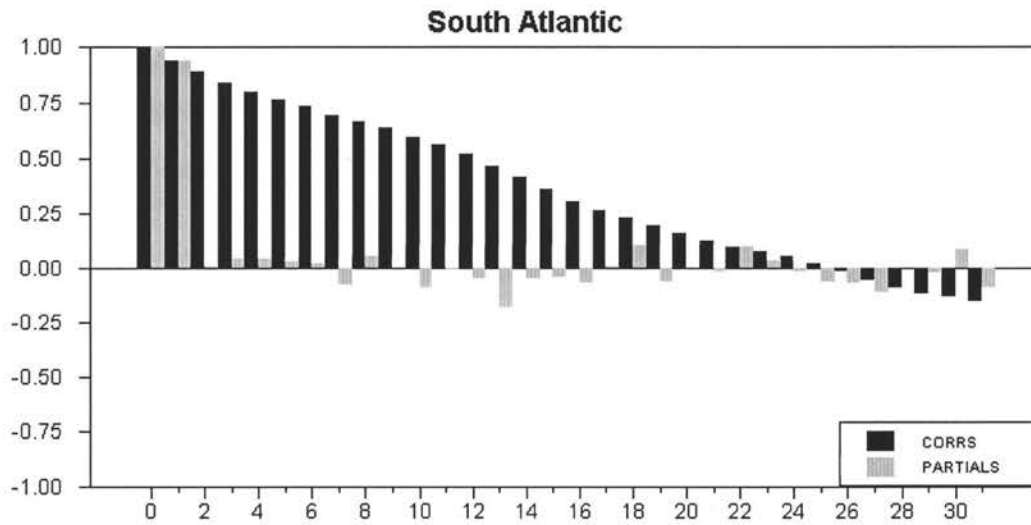


Figure 8: South Atlantic in the Identification Step

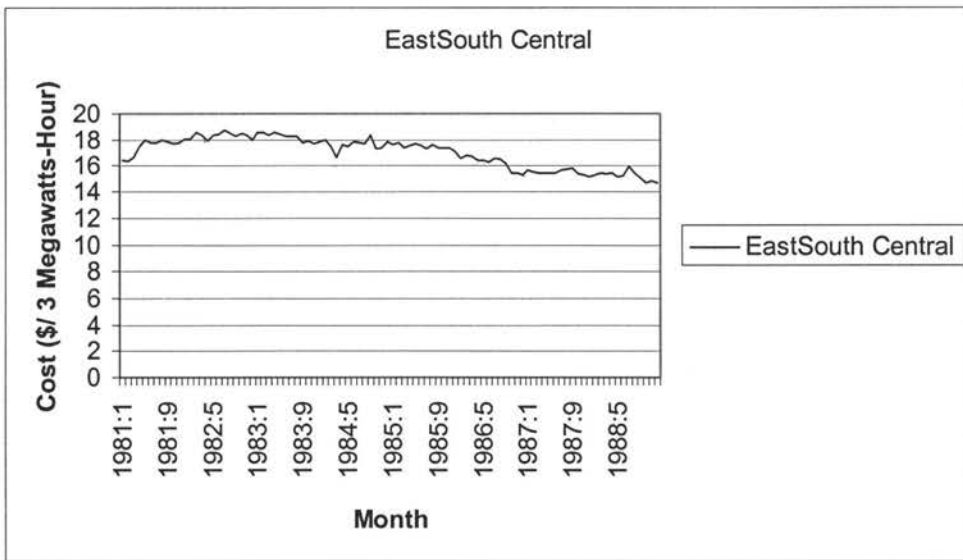


Figure 9: EastSouth Central's Production Cost

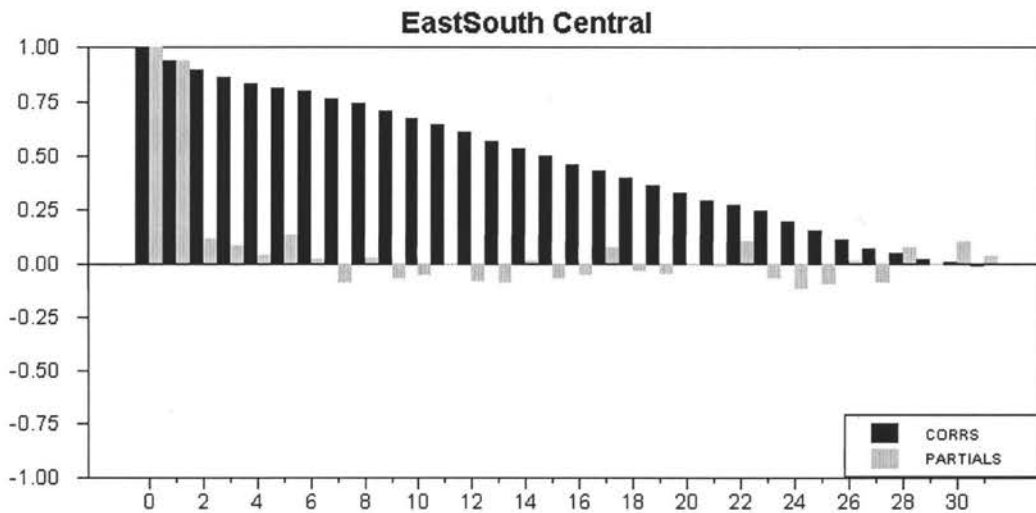


Figure 10: EastSouth Central in the Identification Step

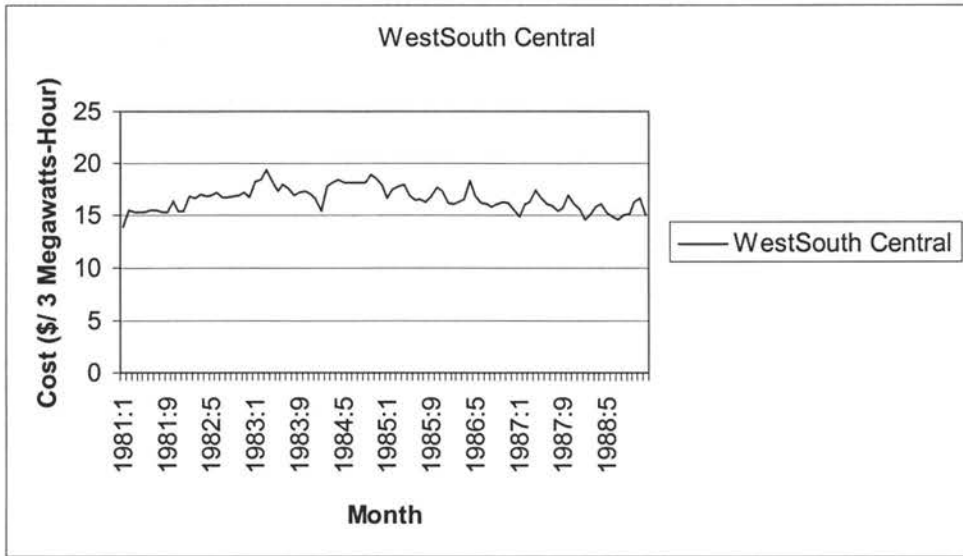


Figure 11: WestSouth Central's Production Cost

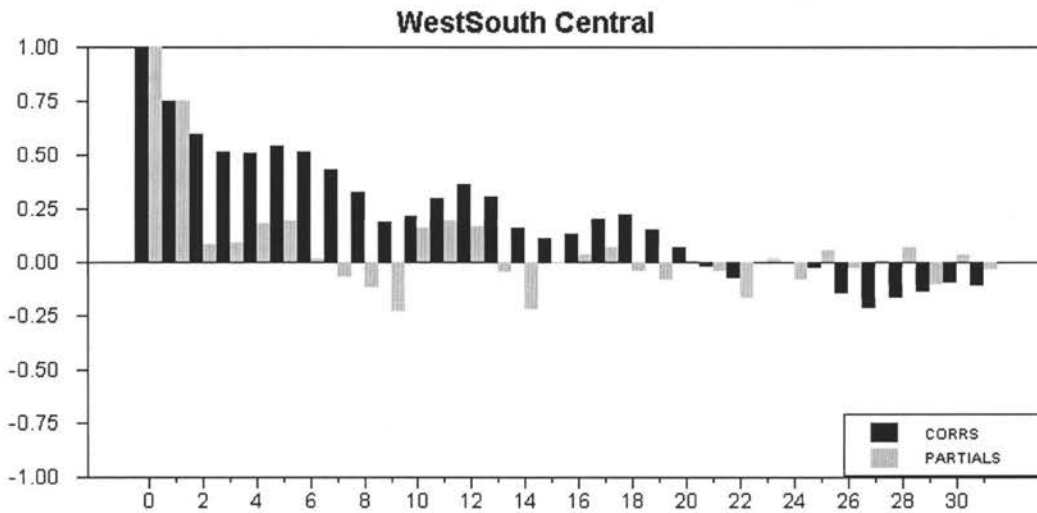


Figure 12: WestSouth Central in the Identification Step

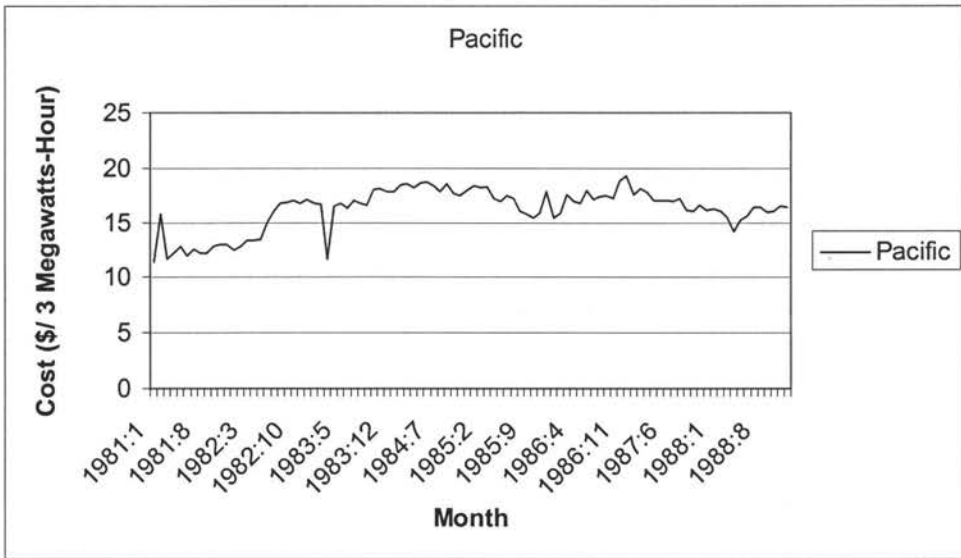


Figure 13: Pacific's Production Cost

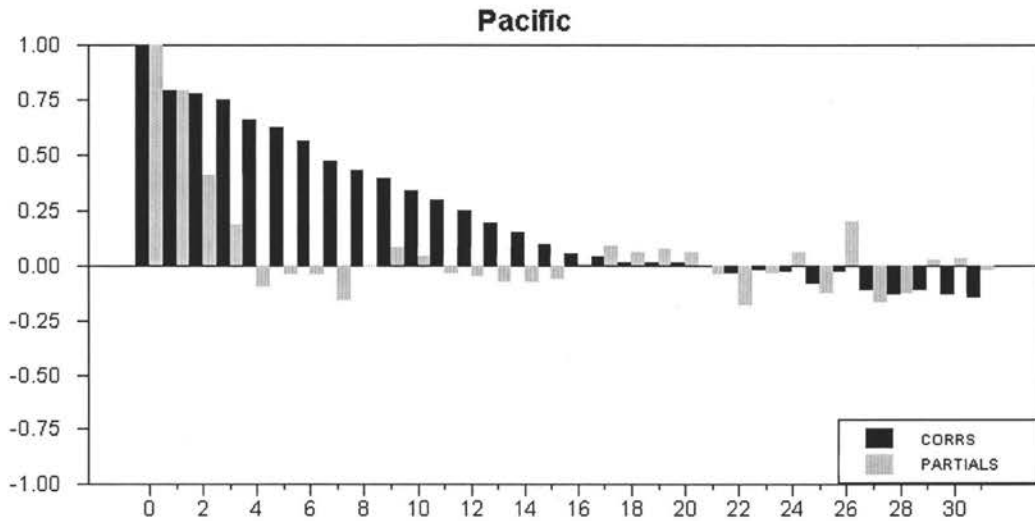


Figure 14: Pacific in the Identification Step

APPENDIX B: A TABLE OF CRITICAL CHI-SQUARE VALUES

Table 1: Chi-squared Table

| Pr d.f. | .250 | .100 | .050 | .025 | .010 | .005 | .001 |
|------------|------|------|------|------|------|------|------|
| 1 | 1.32 | 2.71 | 3.84 | 5.02 | 6.63 | 7.88 | 10.8 |
| 2 | 2.77 | 4.61 | 5.90 | 7.38 | 9.21 | 10.6 | 13.8 |
| 3 | 4.11 | 6.25 | 7.81 | 9.35 | 11.3 | 12.8 | 16.3 |
| 4 | 5.39 | 7.78 | 9.49 | 11.1 | 13.3 | 14.9 | 18.5 |
| 5 | 6.63 | 9.24 | 11.1 | 12.8 | 15.1 | 16.7 | 20.5 |
| 6 | 7.84 | 10.6 | 12.6 | 14.4 | 16.8 | 18.5 | 22.5 |
| 7 | 9.04 | 12.0 | 14.1 | 16.0 | 18.5 | 20.3 | 24.3 |
| 8 | 10.2 | 13.4 | 15.5 | 17.5 | 20.1 | 22.0 | 26.1 |
| 9 | 11.4 | 14.7 | 16.9 | 19.0 | 21.7 | 23.6 | 27.9 |
| 10 | 12.5 | 16.0 | 18.3 | 20.5 | 23.2 | 25.2 | 29.6 |
| 11 | 13.7 | 17.3 | 19.7 | 21.9 | 24.7 | 26.8 | 31.3 |
| 12 | 14.8 | 18.5 | 21.0 | 23.3 | 26.2 | 28.3 | 32.9 |
| 13 | 16.0 | 19.8 | 22.4 | 24.7 | 27.7 | 29.8 | 34.5 |
| 14 | 17.1 | 21.1 | 23.7 | 26.1 | 29.1 | 31.3 | 36.1 |
| 15 | 18.2 | 22.3 | 25.0 | 27.5 | 30.6 | 32.8 | 37.7 |
| 16 | 19.4 | 23.5 | 26.3 | 28.8 | 32.0 | 34.3 | 39.3 |
| 17 | 20.5 | 24.8 | 27.6 | 30.2 | 33.4 | 35.7 | 40.8 |
| 18 | 21.6 | 26.0 | 28.9 | 31.5 | 34.8 | 37.2 | 42.3 |
| 19 | 22.7 | 27.2 | 30.1 | 32.9 | 36.2 | 38.6 | 43.8 |
| 20 | 23.8 | 28.4 | 31.4 | 34.2 | 37.6 | 40.0 | 45.3 |
| 21 | 24.9 | 29.6 | 32.7 | 35.5 | 38.9 | 41.3 | 46.8 |
| 22 | 26.0 | 30.8 | 33.9 | 36.8 | 40.3 | 42.8 | 48.3 |
| 23 | 27.1 | 32.0 | 35.2 | 38.1 | 41.6 | 44.2 | 49.7 |
| 24 | 28.2 | 33.2 | 35.4 | 39.4 | 42.0 | 45.6 | 51.2 |
| 25 | 29.3 | 34.4 | 37.7 | 40.6 | 44.3 | 46.9 | 52.6 |
| 26 | 30.4 | 35.6 | 38.9 | 41.9 | 45.6 | 48.3 | 54.1 |
| 27 | 31.5 | 36.7 | 40.1 | 43.2 | 47.0 | 49.6 | 55.5 |
| 28 | 32.6 | 37.9 | 41.3 | 44.5 | 48.3 | 51.0 | 56.9 |
| 29 | 33.7 | 39.1 | 42.6 | 45.7 | 49.6 | 52.3 | 58.3 |

Source: Ronald J. Wonnacott and Thomas H. Wonnacott, *Econometrics*, 2nd ed., John Wiley & Sons, 1979.

APPENDIX C: JAVASCRIPT PROGRAMS AND SOURCE CODES

Enter number of participating suppliers - Microsoft Internet Explorer provided by America Online

File Edit View Favorites Tools Help

Back Forward Stop Refresh Home Search Favorites History Mail Print Edit Discuss Real.com Messenger ICQ

Address Go

Links

Enter number of suppliers

Enter number of demands

Enter interval of cost production between and

Enter your cost of production

Result

Your R =

Your H =

Your H + R =

$\int c^2 (H+R)dc =$

$B(c) = c +$ $(H(c) + R(c)) =$

Done My Computer

Figure 1: The JavaScript web-based program for 7 suppliers and 2 demands

Enter number of participating suppliers - Microsoft Internet Explorer provided by America Online

File Edit View Favorites Tools Help

Back Forward Stop Refresh Home Search Favorites History Mail Print Edit Discuss Real.com Messenger ICQ

Address Go

Links

Enter number of suppliers

Enter number of demands

Enter interval of cost production between and

Enter your cost of production

Result

Your R =

Your H =

Your H + R =

$\int c^2 (H+R)dc =$

$B(c) = c +$ $(H(c) + R(c)) =$

Done My Computer

Figure 2: The JavaScript web-based program for 7 suppliers and 3 demands

Enter number of participating suppliers - Microsoft Internet Explorer provided by America Online

File Edit View Favorites Tools Help

Back Forward Stop Refresh Home Search Favorites History Mail Print Edit Discuss Real.com Messenger ICQ

Address C:\Thesis\New\Program-R74.html

Links AOL Anywhere AOL Corporate Site AOL Hometown AOL Instant Messenger AOL Mail AOL Search Classifieds Comics Find a Business

Enter number of suppliers

Enter number of demands

Enter interval of cost production between and

Enter your cost of production

Submit Reset

Result

Your R =

Your H =

Your H + R =

$\int_c^{c^2} (H+R)dc =$

Find Optimal Bid

$B(c) = c +$ $/ (H(c) + R(c)) =$

Done My Computer

Figure 3: The JavaScript web-based program for 7 suppliers and 4 demands

Enter number of participating suppliers - Microsoft Internet Explorer provided by America Online

File Edit View Favorites Tools Help

Back Forward Stop Refresh Home Search Favorites History Mail Print Edit Discuss Real.com Messenger ICQ

Address C:\Thesis\New\Program-R75.html

Links AOL Anywhere AOL Corporate Site AOL Hometown AOL Instant Messenger AOL Mail AOL Search Classifieds Comics Find a Business

Enter number of suppliers

Enter number of demands

Enter interval of cost production between and

Enter your cost of production

Submit Reset

Result

Your R =

Your H =

Your H + R =

$\int_c^{c^2} (H+R)dc =$

Find Optimal Bid

$B(c) = c +$ $/ (H(c) + R(c)) =$

Done My Computer

Figure 4: The JavaScript web-based program for 7 suppliers and 5 demands

Table 1: Source code of the program in Figure 1 in Appendix C

```

<html>
<head>
<meta http-equiv="Content-Language" content="en-us">
<meta http-equiv="Content-Type" content="text/html; charset=windows-1252">
<meta name="GENERATOR" content="Microsoft FrontPage 4.0">
<title>Enter number of participating suppliers</title>

<SCRIPT LANGUAGE="JavaScript">

function FindAllResults(form) {

suppliers = eval(form.suppliers.value);
demands  = eval(form.demands.value);
c1      = eval(form.c1.value);
c2      = eval(form.c2.value);
c       = eval(form.c.value);
PrOtherMore = (c2 - c)/(c2-c1);
PrOtherLess = (c-c1)/(c2-c1);
////////////////////////////////////////////////// R's part
YourR =0;
YourH =0;
for (j=0;j<=demands-2;j++) // R's part
    {
    fact1 =1;
    fact2 =1;
    factDiffer =1;
    RtempLess = Math.pow(PrOtherLess,j);
    RtempMore = Math.pow(PrOtherMore,suppliers-1-j);

    for (i=1;i<=suppliers-1; i++) { fact1 = i*fact1; } // Find factorial of n-1
    if (j==0 ) { fact2=1;} // Find factorial of j
    else { for (i=1;i<=j; i++) { fact2 = i*fact2; }}
    for (i=1;i<=suppliers-1-j; i++) { factDiffer = i*factDiffer; } // Find factorial of (n-1-j)
    YourR = YourR + (fact1/fact2/factDiffer)*RtempLess*RtempMore;
    } // end R's part

////////////////////////////////////////////////// H's part
factH1 =1;
factH2 =1;
factHDiffer =1;
HtempLess = Math.pow(PrOtherLess,demands-1);
HtempMore = Math.pow(PrOtherMore,suppliers-demands);

for (i=1;i<=suppliers-1; i++) { factH1 = i*factH1; }
for (j=0;j<=demands-1;j++)
    {
    if (j==0 ) { factH2=1;} // Find factorial of j
    else { factH2 = j*factH2; }
}

```

```

    }
    for (i=1;i<=suppliers-demands; i++) { factHDiffer = i*factHDiffer; } // factorial of (n-1-j)
    YourH = (factH1/factH2/factHDiffer)*HtempLess*HtempMore;
    //////////////////////////////////////////////////////////////////// End H's part
    form.R.value = YourR;
    form.H.value = YourH;
    form.HandR.value = YourR + YourH;
    //////////////////////////////////////////////////////////////////// H + R with integrate
    textTemp = "";
    UpperB = c2-c1;
    LowerB = c-c1;
    form.C1C2devide.value = 1/(Math.pow(c2-c1,suppliers-1));

    for (j=0;j<=demands-1;j++)
    {
        fact1 = 1;
        fact2 = 1;
        factDiffer = 1;

        for (i=1;i<=suppliers-1; i++) { fact1 = i*fact1; } // Find factorial of n-1
        if (j==0 ) { fact2=1;} // Find factorial of j
        else { for (i=1;i<=j; i++) { fact2 = i*fact2; }}
        for (i=1;i<=suppliers-1-j; i++) { factDiffer = i*factDiffer; } // Find factorial of (n-1-j)
        textTemp = textTemp + "(" + (fact1/fact2/factDiffer) + ")" + "*" + "x" + j + "(" +
        UpperB + "-x)^" + (suppliers-1-j);
        if (j<=demands-2) { textTemp = textTemp + " + "; }

    }

    textTemp = textTemp + " ==> integrate from " + LowerB + " to " + UpperB ;
    form.long.value = textTemp;
    //////////////////////////////////////////////////////////////////
    FirstValue = 531441*UpperB - 32805*(Math.pow(UpperB,3)) + 7290*(Math.pow(UpperB,4))
    - 729*(Math.pow(UpperB,5)) + 36*(Math.pow(UpperB,6)) - (5*(Math.pow(UpperB,7)))/7;
    SecondValue = 531441*LowerB - 32805*(Math.pow(LowerB,3)) +
    7290*(Math.pow(LowerB,4)) - 729*(Math.pow(LowerB,5)) + 36*(Math.pow(LowerB,6)) -
    (5*(Math.pow(LowerB,7)))/7;
    form.RandHafterIntegrate.value = FirstValue - SecondValue;
    //////////////////////////////////////////////////////////////////
}

function FinalResult(form) {

suppliers = eval(form.suppliers.value);
demands = eval(form.demands.value);
c1 = eval(form.c1.value);
c2 = eval(form.c2.value);
c = eval(form.c.value);
//AfterIntegrate = eval(form.RandHafterIntegrate.value);

```

```

RepeatH = eval(form.H.value);
RepeatR = eval(form.R.value);
Both = RepeatH + RepeatR;
RandHafterIntegrate = eval(form.RandHafterIntegrate.value);
form.OptimalBid.value = c + (1/(Math.pow(c2-c1,suppliers1))*RandHafterIntegrate)/Both;
}

</SCRIPT>
</head>

<body>
<form method="POST" NAME="theForm">
  <p>Enter number of suppliers <input type="text" name="suppliers" size="6"></p>
  <p>Enter number of demands <input type="text" name="demands" size="6"></p>
  <p>Enter interval of cost production between <input type="text" name="c1" size="6">
  and <input type="text" name="c2" size="6"></p>
  <p>Enter your cost of production <input type="text" name="c" size="6"></p>
  <p><input type="button" value="Submit" name="B1"
onClick="FindAllResults(this.form)"><input type="reset" value="Reset" name="B2"></p>
  <hr>
  <p align="center"><b><u><font size="5">Result</font></u></b></p>
  <p>Your R = <input type="text" name="R" size="20"></p>
  <p>Your H = <input type="text" name="H" size="20"></p>
  <p>Your H + R = <input type="text" name="HandR" size="20"></p>
  <p><span style="font-size:14.0pt;mso-bidi-font-size:12.0pt;
font-family:&quot;Times New Roman&quot;;mso-fareast-font-family:&quot;Times New
Roman&quot;;
mso-ansi-language:EN-US;mso-fareast-language:EN-US;mso-bidi-language:AR-
SA">&#8747;<sub>c</sub></span><span style="font-size: 14.0pt; mso-bidi-font-size:
12.0pt; font-family: Times New Roman; mso-fareast-font-family: Times New Roman; mso-
ansi-language: EN-US; mso-fareast-language: EN-US; mso-bidi-language: AR-
SA"><sup>c2</sup></span>(H+R)dc
= <input type="text" name="C1C2devide" size="10"><span style="font-size: 14.0pt; mso-
bidi-font-size: 12.0pt"><sup>*
</sup></span> <input type="text" name="long" size="94"></p>
  <p><input type="button" value="Submit" name="B3" onClick="FinalResult(this.form)"></p>
  <p>B(c) = c + <input type="text" name="RandHafterIntegrate" size="10">/(H(c) +
R(c)) = <input type="text" name="OptimalBid" size="41"></p>
</form>

</body>
</html>

```

Table 2: Source code of the program in Figure 2 in Appendix C

```

<html>
<head>
<meta http-equiv="Content-Language" content="en-us">
<meta http-equiv="Content-Type" content="text/html; charset=windows-1252">
<meta name="GENERATOR" content="Microsoft FrontPage 4.0">
<title>Enter number of participating suppliers</title>

<SCRIPT LANGUAGE="JavaScript">

function FindAllResults(form) {

suppliers = eval(form.suppliers.value);
demands = eval(form.demands.value);
c1 = eval(form.c1.value);
c2 = eval(form.c2.value);
c = eval(form.c.value);
PrOtherMore = (c2 - c)/(c2-c1);
PrOtherLess = (c-c1)/(c2-c1);
//////////////////////////////////// R's part
YourR =0;
YourH =0;
for (j=0;j<=demands-2;j++) // R's part
    {
    fact1 =1;
    fact2 =1;
    factDiffer =1;
    RtempLess = Math.pow(PrOtherLess,j);
    RtempMore = Math.pow(PrOtherMore,suppliers-1-j);

    for (i=1;j<=suppliers-1; i++) { fact1 = i*fact1; } // Find factorial of n-1
    if (j==0 ) { fact2=1;} // Find factorial of j
    else { for (i=1;j<=j; i++) { fact2 = i*fact2; } }
    for (i=1;j<=suppliers-1-j; i++) { factDiffer = i*factDiffer; } // Find factorial of (n-1-j)
    YourR = YourR + (fact1/fact2/factDiffer)*RtempLess*RtempMore;
    } // end R's part
//////////////////////////////////// H's part
factH1 =1;
factH2 =1;
factHDiffer =1;
HtempLess = Math.pow(PrOtherLess,demands-1);
HtempMore = Math.pow(PrOtherMore,suppliers-demands);
for (i=1;j<=suppliers-1; i++) { factH1 = i*factH1; }
for (j=0;j<=demands-1;j++)
    {
    if (j==0 ) { factH2=1;} // Find factorial of j
    else { factH2 = j*factH2; }
    }
for (i=1;j<=suppliers-demands; i++) { factHDiffer = i*factHDiffer; } // factorial of (n-1-j)

```



```

YourH = (factH1/factH2/factHDiffer)*HtempLess*HtempMore;
////////// End H's part
form.R.value = YourR;
form.H.value = YourH;
form.HandR.value = YourR + YourH;
////////// H + R with integrate
textTemp = "";
UpperB = c2-c1;
LowerB = c-c1;
form.C1C2devide.value = 1/(Math.pow(c2-c1,suppliers-1));
for (j=0;j<=demands-1;j++)
{
    fact1 =1;
    fact2 =1;
    factDiffer =1;

    for (i=1;j<=suppliers-1; i++) { fact1 = i*fact1; } // Find factorial of n-1
    if (j==0) { fact2=1;} // Find factorial of j
    else { for (i=1;i<=j; i++) { fact2 = i*fact2; }}
    for (i=1;i<=suppliers-1-j; i++) { factDiffer = i*factDiffer; } // Find factorial of (n-1-j)
    textTemp = textTemp + "(" + (fact1/fact2/factDiffer) + ")" + "*" + "x" + j + "(" +
UpperB + "-x)" + (suppliers-1-j);
    if (j<=demands-2) { textTemp = textTemp + " + "; }
}

textTemp = textTemp + " ==> integrate from " + LowerB + " to " + UpperB ;
form.long.value = textTemp;
/////
FirstValue = 531441*UpperB - 3645*(Math.pow(UpperB,4)) + 729*(Math.pow(UpperB,5)) -
54*(Math.pow(UpperB,6)) + (10*(Math.pow(UpperB,7)))/7;
SecondValue = 531441*LowerB - 3645*(Math.pow(LowerB,4)) + 729*(Math.pow(LowerB,5))
- 54*(Math.pow(LowerB,6)) + (10*(Math.pow(LowerB,7)))/7;
form.RandHafterIntegrate.value = FirstValue - SecondValue;
/////
}
function FinalResult(form) {

suppliers = eval(form.suppliers.value);
demands = eval(form.demands.value);
c1 = eval(form.c1.value);
c2 = eval(form.c2.value);
c = eval(form.c.value);
//AfterIntegrate = eval(form.RandHafterIntegrate.value);
RepeatH = eval(form.H.value);
RepeatR = eval(form.R.value);
Both = RepeatH + RepeatR;
RandHafterIntegrate = eval(form.RandHafterIntegrate.value);
form.OptimalBid.value = c + (1/(Math.pow(c2-c1,suppliers1))*RandHafterIntegrate)/Both;
}

```

```

</SCRIPT>
</head>

<body>
<form method="POST" NAME="theForm">
  <p>Enter number of suppliers <input type="text" name="suppliers" size="6"></p>
  <p>Enter number of demands <input type="text" name="demands" size="6"></p>
  <p>Enter interval of cost production between <input type="text" name="c1" size="6">
  and <input type="text" name="c2" size="6"></p>
  <p>Enter your cost of production <input type="text" name="c" size="6"></p>
  <p><input type="button" value="Submit" name="B1"
onClick="FindAllResults(this.form)"><input type="reset" value="Reset" name="B2"></p>
  <hr>
  <p align="center"><b><u><font size="5">Result</font></u></b></p>
  <p>Your R = <input type="text" name="R" size="20"></p>
  <p>Your H = <input type="text" name="H" size="20"></p>
  <p>Your H + R = <input type="text" name="HandR" size="20"></p>
  <p><span style="font-size:14.0pt;mso-bidi-font-size:12.0pt;
font-family:&quot;Times New Roman&quot;;mso-fareast-font-family:&quot;Times New
Roman&quot;;
mso-ansi-language:EN-US;mso-fareast-language:EN-US;mso-bidi-language:AR-
SA">&#8747;<sub>c</sub></span><span style="font-size: 14.0pt; mso-bidi-font-size:
12.0pt; font-family: Times New Roman; mso-fareast-font-family: Times New Roman; mso-
ansi-language: EN-US; mso-fareast-language: EN-US; mso-bidi-language: AR-
SA"><sup>c2</sup></span>(H+R)dc
  = <input type="text" name="C1C2divide" size="10"><span style="font-size: 14.0pt; mso-
bidi-font-size: 12.0pt"><sup>*
  </sup></span> <input type="text" name="long" size="94"></p>
  <p><input type="button" value="Find Optimal Bid" name="B3"
onClick="FinalResult(this.form)"></p>
  <p>B(c) = c + <input type="text" name="RandHafterIntegrate" size="10">/(H(c) +
  R(c)) = <input type="text" name="OptimalBid" size="41"></p>
</form>

</body>
</html>

```

Table 3: Source code of the program in Figure 3 in Appendix C

```

<html>
<head>
<meta http-equiv="Content-Language" content="en-us">
<meta http-equiv="Content-Type" content="text/html; charset=windows-1252">
<meta name="GENERATOR" content="Microsoft FrontPage 4.0">
<title>Enter number of participating suppliers</title>

<SCRIPT LANGUAGE="JavaScript">

function FindAllResults(form) {

suppliers = eval(form.suppliers.value);
demands  = eval(form.demands.value);
c1       = eval(form.c1.value);
c2       = eval(form.c2.value);
c        = eval(form.c.value);
PrOtherMore = (c2 - c)/(c2-c1);
PrOtherLess = (c-c1)/(c2-c1);
//////////////////////////////////////////////////////////////// R's part
YourR =0;
YourH =0;
for (j=0;j<=demands-2;j++) // R's part
    {
    fact1 =1;
    fact2 =1;
    factDiffer =1;
    RtempLess = Math.pow(PrOtherLess,j);
    RtempMore = Math.pow(PrOtherMore,suppliers-1-j);

    for (i=1;i<=suppliers-1; i++) { fact1 = i*fact1; } // Find factorial of n-1
    if (j==0 ) { fact2=1;} // Find factorial of j
    else { for (i=1;i<=j; i++) { fact2 = i*fact2; }}
    for (i=1;i<=suppliers-1-j; i++) { factDiffer = i*factDiffer; } // Find factorial of (n-1-j)
    YourR = YourR + (fact1/fact2/factDiffer)*RtempLess*RtempMore;
    } // end R's part
//////////////////////////////////////////////////////////////// H's part
factH1 =1;
factH2 =1;
factHDiffer =1;
HtempLess = Math.pow(PrOtherLess,demands-1);
HtempMore = Math.pow(PrOtherMore,suppliers-demands);

for (i=1;i<=suppliers-1; i++) { factH1 = i*factH1; }
for (j=0;j<=demands-1;j++)
    {
    if (j==0 ) { factH2=1;} // Find factorial of j
    else { factH2 = j*factH2; }
    }
}

```

```

for (i=1;i<=suppliers-demands; i++) { factHDiffer = i*factHDiffer; } // Find factorial of (n-1-j)
YourH = (factH1/factH2/factHDiffer)*HtempLess*HtempMore;
//////////////////////////////////// End H's part
form.R.value = YourR;
form.H.value = YourH;
form.HandR.value = YourR + YourH;

//////////////////////////////////// H + R with integrate
textTemp = "";
UpperB = c2-c1;
LowerB = c-c1;
form.C1C2devide.value = 1/(Math.pow(c2-c1,suppliers-1));
for (j=0;j<=demands-1;j++)
{
    fact1 = 1;
    fact2 = 1;
    factDiffer = 1;

    for (i=1;i<=suppliers-1; i++) { fact1 = i*fact1; } // Find factorial of n-1
    if (j==0) { fact2=1;} // Find factorial of j
    else { for (i=1;i<=j; i++) { fact2 = i*fact2; }}
    for (i=1;i<=suppliers-1-j; i++) { factDiffer = i*factDiffer; } // Find factorial of (n-1-j)
    textTemp = textTemp + "(" + (fact1/fact2/factDiffer) + ")" + "*" + "x" + j + "(" +
UpperB + "-x)" + (suppliers-1-j);
    if (j<=demands-2) { textTemp = textTemp + " + "; }

}
textTemp = textTemp + " ==> integrate from " + LowerB + " to " + UpperB ;
form.long.value = textTemp;
/////
FirstValue = 531441*UpperB - 243*(Math.pow(UpperB,5)) + 36*(Math.pow(UpperB,6)) -
(10*(Math.pow(UpperB,7)))/7;
SecondValue = 531441*LowerB - 243*(Math.pow(LowerB,5)) + 36*(Math.pow(LowerB,6)) -
(10*(Math.pow(LowerB,7)))/7;
form.RandHafterIntegrate.value = FirstValue - SecondValue;
/////
}

function FinalResult(form) {

suppliers = eval(form.suppliers.value);
demands = eval(form.demands.value);
c1 = eval(form.c1.value);
c2 = eval(form.c2.value);
c = eval(form.c.value);
//AfterIntegrate = eval(form.RandHafterIntegrate.value);
RepeatH = eval(form.H.value);
RepeatR = eval(form.R.value);

```

```

Both = RepeatH + RepeatR;

RandHafterIntegrate = eval(form.RandHafterIntegrate.value);
form.OptimalBid.value = c + (1/(Math.pow(c2-c1,suppliers1))*RandHafterIntegrate)/Both;
}
</SCRIPT>
</head>

<body>
<form method="POST" NAME="theForm">
  <p>Enter number of suppliers <input type="text" name="suppliers" size="6"></p>
  <p>Enter number of demands <input type="text" name="demands" size="6"></p>
  <p>Enter interval of cost production between <input type="text" name="c1" size="6">
  and <input type="text" name="c2" size="6"></p>
  <p>Enter your cost of production <input type="text" name="c" size="6"></p>
  <p><input type="button" value="Submit" name="B1"
onClick="FindAllResults(this.form)"><input type="reset" value="Reset" name="B2"></p>
  <hr>
  <p align="center"><b><u><font size="5">Result</font></u></b></p>
  <p>Your R = <input type="text" name="R" size="20"></p>
  <p>Your H = <input type="text" name="H" size="20"></p>
  <p>Your H + R = <input type="text" name="HandR" size="20"></p>
  <p><span style="font-size:14.0pt;mso-bidi-font-size:12.0pt;
font-family:&quot;Times New Roman&quot;;mso-fareast-font-family:&quot;Times New
Roman&quot;;
mso-ansi-language:EN-US;mso-fareast-language:EN-US;mso-bidi-language:AR-
SA">&#8747;<sub>c</sub></span><span style="font-size: 14.0pt; mso-bidi-font-size:
12.0pt; font-family: Times New Roman; mso-fareast-font-family: Times New Roman; mso-
ansi-language: EN-US; mso-fareast-language: EN-US; mso-bidi-language: AR-
SA"><sup>c2</sup></span>(H+R)dc
= <input type="text" name="C1C2devide" size="10"><span style="font-size: 14.0pt; mso-
bidi-font-size: 12.0pt"><sup>*
</sup></span> <input type="text" name="long" size="94"></p>
  <p><input type="button" value="Find Optimal Bid" name="B3"
onClick="FinalResult(this.form)"></p>
  <p>B(c) = c + <input type="text" name="RandHafterIntegrate" size="10">/(H(c) +
R(c)) = <input type="text" name="OptimalBid" size="41"></p>
</form>

</body>
</html>

```

Table 4: Source code of the program in Figure 4 in Appendix C

```

<html>
<head>
<meta http-equiv="Content-Language" content="en-us">
<meta http-equiv="Content-Type" content="text/html; charset=windows-1252">
<meta name="GENERATOR" content="Microsoft FrontPage 4.0">
<title>Enter number of participating suppliers</title>

<SCRIPT LANGUAGE="JavaScript">

function FindAllResults(form) {

suppliers = eval(form.suppliers.value);
demands = eval(form.demands.value);
c1 = eval(form.c1.value);
c2 = eval(form.c2.value);
c = eval(form.c.value);
PrOtherMore = (c2 - c)/(c2-c1);
PrOtherLess = (c-c1)/(c2-c1);
////////////////////////////////////////////////////////////////// R's part
YourR =0;
YourH =0;
for (j=0;j<=demands-2;j++) // R's part
{
fact1 =1;
fact2 =1;
factDiffer =1;
RtempLess = Math.pow(PrOtherLess,j);
RtempMore = Math.pow(PrOtherMore,suppliers-1-j);

for (i=1;i<=suppliers-1; i++) { fact1 = i*fact1; } // Find factorial of n-1
if (j==0 ) { fact2=1;} // Find factorial of j
else { for (i=1;i<=j; i++) { fact2 = i*fact2; }}
for (i=1;i<=suppliers-1-j; i++) { factDiffer = i*factDiffer; } // Find factorial of (n-1-j)
YourR = YourR + (fact1/fact2/factDiffer)*RtempLess*RtempMore;
} // end R's part
////////////////////////////////////////////////////////////////// H's part
factH1 =1;
factH2 =1;
factHDiffer =1;
HtempLess = Math.pow(PrOtherLess,demands-1);
HtempMore = Math.pow(PrOtherMore,suppliers-demands);
for (i=1;i<=suppliers-1; i++) { factH1 = i*factH1; }
for (j=0;j<=demands-1;j++)
{
if (j==0 ) { factH2=1;} // Find factorial of j
else { factH2 = j*factH2; }
}
}

```

```

for (i=1;i<=suppliers-demands; i++) { factHDiffer = i*factHDiffer; } // Find factorial of (n-1-j)
YourH = (factH1/factH2/factHDiffer)*HtempLess*HtempMore;
//////////////////////////////////// End H's part
form.R.value = YourR;
form.H.value = YourH;
form.HandR.value = YourR + YourH;
//////////////////////////////////// H + R with integrate
textTemp = "";
UpperB = c2-c1;
LowerB = c-c1;
form.C1C2devide.value = 1/(Math.pow(c2-c1,suppliers-1));

for (j=0;j<=demands-1;j++)
{
    fact1 =1;
    fact2 =1;
    factDiffer =1;

    for (i=1;i<=suppliers-1; i++) { fact1 = i*fact1; } // Find factorial of n-1
    if (j==0 ) { fact2=1;} // Find factorial of j
    else { for (i=1;i<=j; i++) { fact2 = i*fact2; }}
    for (i=1;i<=suppliers-1-j; i++) { factDiffer = i*factDiffer; } // Find factorial of (n-1-j)
    textTemp = textTemp + "(" + (fact1/fact2/factDiffer) + ")" + "*" + "x" + j + "(" +
UpperB + "-x)" + (suppliers-1-j);
    if (j<=demands-2) { textTemp = textTemp + " + "; }

}

textTemp = textTemp + " ==> integrate from " + LowerB + " to " + UpperB ;
form.long.value = textTemp;
/////
FirstValue = 531441*UpperB - 9*(Math.pow(UpperB,6)) + (5*(Math.pow(UpperB,7)))/7;
SecondValue = 531441*LowerB - 9*(Math.pow(LowerB,6)) + (5*(Math.pow(LowerB,7)))/7;
form.RandHafterIntegrate.value = FirstValue - SecondValue;
/////
}

function FinalResult(form) {

suppliers = eval(form.suppliers.value);
demands = eval(form.demands.value);
c1 = eval(form.c1.value);
c2 = eval(form.c2.value);
c = eval(form.c.value);
//AfterIntegrate = eval(form.RandHafterIntegrate.value);
RepeatH = eval(form.H.value);
RepeatR = eval(form.R.value);
Both = RepeatH + RepeatR;

```

```

RandHafterIntegrate = eval(form.RandHafterIntegrate.value);
form.OptimalBid.value = c + (1/(Math.pow(c2-c1,suppliers1))*RandHafterIntegrate)/Both;
}
</SCRIPT>
</head>

<body>
<form method="POST" NAME="theForm">
  <p>Enter number of suppliers <input type="text" name="suppliers" size="6"></p>
  <p>Enter number of demands <input type="text" name="demands" size="6"></p>
  <p>Enter interval of cost production between <input type="text" name="c1" size="6">
  and <input type="text" name="c2" size="6"></p>
  <p>Enter your cost of production <input type="text" name="c" size="6"></p>
  <p><input type="button" value="Submit" name="B1"
onClick="FindAllResults(this.form)"><input type="reset" value="Reset" name="B2"></p>
  <hr>
  <p align="center"><b><u><font size="5">Result</font></u></b></p>
  <p>Your R = <input type="text" name="R" size="20"></p>
  <p>Your H = <input type="text" name="H" size="20"></p>
  <p>Your H + R = <input type="text" name="HandR" size="20"></p>
  <p><span style="font-size:14.0pt;mso-bidi-font-size:12.0pt;
font-family:"Times New Roman";mso-fareast-font-family:"Times New
Roman";
mso-ansi-language:EN-US;mso-fareast-language:EN-US;mso-bidi-language:AR-
SA">&#8747;<sub>c</sub></span><span style="font-size: 14.0pt; mso-bidi-font-size:
12.0pt; font-family: Times New Roman; mso-fareast-font-family: Times New Roman; mso-
ansi-language: EN-US; mso-fareast-language: EN-US; mso-bidi-language: AR-
SA"><sup>c2</sup></span>(H+R)dc
= <input type="text" name="C1C2divide" size="10"><span style="font-size: 14.0pt; mso-
bidi-font-size: 12.0pt"><sup>*
</sup></span> <input type="text" name="long" size="94"></p>
  <p><input type="button" value="Find Optimal Bid" name="B3"
onClick="FinalResult(this.form)"></p>
  <p>B(c) = c + <input type="text" name="RandHafterIntegrate" size="10">/H(c) +
  R(c) = <input type="text" name="OptimalBid" size="41"></p>
</form>

</body>
</html>

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